

Title of manuscript:

“Assigning Pastors to Parishes: An Optimization Model”

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Abstract

Pastors in a particular religious diocese in Canada are required to make monthly visits to parishes. Previously, each year's visit schedule was manually constructed. We report on the development of an analytical model to optimize the assignments. Through the use of our approach, the diocese has reduced its annual travel reimbursement costs. It is planning to make permanent use of this optimization model in identifying monthly visit schedules.

Introduction

Decision-makers use assignment models to optimize resource allocation. A variety of successful applications have been reported in the literature, including medical residents to hospitals (Roth, 1986), loaner equipment to customers experiencing downtime (Karmarkar and Kubat, 1983), Navy personnel to jobs (Liang and Thompson, 1987; Blanco and Hillery, 1994), law students to employment fair interview slots (Bartholdi III and McCroan, 1990), buyer-seller meetings to trade show appointment periods (Ernst, Mills and Welgama, 2003), female collegiate students to sorority organizations (Mongell and Roth, 1991) and flight training time blocks to aeronautical students (Bazargan-Lari, 2004).

In this paper, we develop an integer optimization model to make cost-effective assignments in a non-profit context. Specifically, we assign monthly visits of religious pastors to parishes. The religious organization explored in this paper includes a large diocese in Saskatchewan, Canada with ten respective parishes (also called units). Figure 1 shows a map of the diocese area with the parish locations circled (the city of Saskatoon near the bottom of the map includes four parishes).

===== insert Figure 1 about here =====

The 11 pastors in this diocese are required to make periodic visits to units in order to meet with parish members, provide religious instruction, speak at church services, and train parish leaders. The pastors are un-paid volunteers who maintain regular careers during the rest of the week. The only compensation received for their parish visits involves a reimbursement of round-trip travel costs. Table 1 lists the first names of each pastor and indicates those who reside outside of Saskatoon (the notion of stewardship parishes will be discussed in the model section).

===== insert Table 1 about here =====

In the past, a well-meaning diocese volunteer had manually created each year's speaking schedule. Regrettably, such an approach was fraught with complexities for the scheduler and frustration by all parties involved (the scheduler and pastors). For example, it was not unusual for a speaking assignment to include a specific pastor visiting the same parish multiple times within a year, a situation that did not provide the variety of speaking engagements the diocese desired for their pastors. In addition, the scheduler frequently devoted several hours and numerous iterations to creating a schedule. Despite his best efforts, no one ever seemed completely satisfied with the final product.

This frustration – combined with the tough economic times experienced in late 2008 – fueled the desire to seek a better solution to the assignment problem. We note here that the amount of dollars devoted to annual pastor travel reimbursement costs is not strikingly large; indeed, it is measured in the thousands of dollars. However, this non-profit organization wanted to be as frugal as possible in the allocation of its resources. Any budget funds saved would be beneficial to the rest of the religious and charitable

work in the diocese. Through a personal contact with one of the pastors, we were approached about providing an analytical solution to this assignment problem.

The remainder of our paper proceeds as follows. The next section describes the development of our optimization model. We then provide model results, after which we conclude the paper with some summary comments.

Method

The optimization model to assign monthly visits of pastors to respective parishes is of the following form:

$$\text{MIN } 0.42 \sum_{t=1}^{11} \sum_{j=1}^{10} \sum_{k=1}^{12} D_{tj} X_{tjk}$$

subject to:

$$(1) \quad \sum_j X_{tjk} \leq 1 \quad \forall t, k$$

$$(2) \quad \sum_j \sum_k X_{tjk} \leq 3 \quad \forall t$$

$$(3) \quad \sum_j \sum_k X_{tjk} \geq 3 \quad \forall t$$

$$(4) \quad \sum_t X_{tjk} = 1 \quad \forall j \in J_1; k \in \text{Quarter}$$

$$(5) \quad \sum_t X_{tjk} = 1 \quad \forall j \in J_2; k \in \text{2 mth pair}$$

$$(6) \quad \sum_t \sum_k X_{tjk} = 10 \quad \forall j \in J_3$$

$$(7) \quad \sum_j X_{tjk} = 0 \quad \forall t, k \text{ holiday combinations}$$

$$(8) \quad \sum_k X_{ijk} = 1 \quad \forall i, j \text{ stewardship combinations}$$

$$(9) \quad \sum_k X_{ijk} \leq 1 \quad \forall i, j$$

$$X_{ijk} \in \{0,1\}$$

In a typical fashion to other types of assignment models, we use a “three-subscript” decision variable to represent the visits. Specifically, we have:

$$X_{ijk} = 1 \text{ if pastor } i \text{ visits parish } j \text{ in month } k$$

$$X_{ijk} = 0 \text{ otherwise}$$

Pastors are re-imbursed for their (round-trip) travel at a rate of \$0.42 per kilometer. Letting D_{ij} represent the round-trip distance between pastor i 's home community and the location of parish j , our objective function calculates the total yearly travel cost for this particular diocese.

After speaking with diocese authorities, we learned of the various policies and regulations governing the assignment of pastors to parishes. We note that some of these rules were violated in the previous years' manual schedules; however, church authorities maintained that they desired a set of assignments that observed the constraints depicted in our optimization model. Constraint (1) restricts each pastor to at most one parish visit per month. Constraints (2) and (3) provide lower and upper bounds, respectively, on the annual number of visits per pastor. These constraints were included so as to ensure some degree of equity in annual assignments. Had this constraint not been included, then a cost-minimizing solution would provide those pastors residing in more remote communities almost no monthly assignments.

For constraints (4) through (6), we recognize that specific parishes within this diocese require a certain annual number of pastor visits. Three units in Saskatoon (1st, 2nd and 3rd) need one visit per quarter. For ease of notation, we labeled these three units as J_1 . Constraint (4) ensures that each of these parishes is visited quarterly.

The Saskatoon 4th, Meadow Lake, Kindersley, Prince Albert and Flin Flon parishes each require one visit every other month. Identifying these units as J_2 , we use constraint (5) to assign a sufficient number of pastors to these parishes. Diocese authorities wanted the Melfort and North Battleford parishes (represented by J_3) to receive 10 visits annually. Constraint (6) permits this number of assignments.

Recall that pastors are lay ministers who hold regular occupations during the week. As such, pastors may seek holiday months during particular portions of the year when they would be unavailable for travel assignments. We use constraint (7) to guarantee that pastors are not assigned to visit a parish in accordance with specific holiday requests.

As indicated in Table 1, nearly all of the pastors have specific parishes over which they have particular stewardships. These stewardships were instituted to maintain a connection between the given pastors and parishes. Should the parish require training or instruction on religious topics, the assigned pastor would be responsible for such work. Constraint (8) guarantees that pastors visit their stewardship units once annually.

Finally, to obviate the dilemma of assigning a given pastor to the same parish multiple times in year, we use constraint (9) to assure that each pastor can visit a particular parish at most once annually.

Results

We formulated and solved our integer programming optimization model using *Premium Solver* on an IBM desktop computer with an Intel Pentium D processor. This particular package works within a spreadsheet environment. Our success with such software further demonstrates the applicability of spreadsheet models in exploring real-world analytical problems (Albright, Winston and Zappe (2003) and Ragsdale (2004) promote spreadsheets in model-building activities).

The optimizer identified a least-cost set of monthly assignments in a few seconds. The solution it proposed for the 2009 travel schedule reduced costs from those experienced with the manual schedules. For example, in 2007 this diocese incurred a travel reimbursement cost of \$13,016. In 2008, the costs climbed to \$13,886. The use of our model reduced the 2009 costs to \$8,989, a reduction of 35.3% in reimbursed expenses. A graphical comparison of the annual costs is offered in Figure 2.

===== insert Figure 2 about here =====

The diocese was quite pleased with these results and adopted the proposed assignment schedule in 2009. It is planning to use the results from this optimization model annually in order to determine the most cost-effective set of pastor visits to respective parishes.

Conclusions

Our paper has described the development of an optimization model to determine the least-cost annual assignment of pastors to parishes. Using a spreadsheet optimizer, we were able to model the particular restrictions desired by the diocese and produce a

solution that reduced costs considerably. The fact that the savings were created during a period of tough economic times further impressed the diocese authorities. Indeed, the diocese is planning to make permanent use of this approach in the annual scheduling of its pastor visits.

Besides the savings in travel reimbursement costs, our model produces schedules in a relatively quick fashion. It has eliminated the hassle and trouble experienced by the scheduler in manually crafting each year's set of assignments. No longer are the parties frustrated by the schedule, and there is a greater sense of equity and fairness in the final product.

Given that the pastors hold regular careers during the week, it is not uncommon for a pastor to relocate outside of the diocese during a calendar year. This necessitates identifying a new pastor to take the relocated pastor's place. Should this happen, one could "lock in" those visits that have already transpired and re-run the optimization model to determine the best set of assignments for the remainder of the year. This provides the diocese with the opportunity to continue to be as cost-effective as possible, even if new pastors are appointed during a year.

Admittedly, our approach is not a complex operations research model. It has not led to an organization saving millions of dollars. Instead, we built an optimization model in Excel to cut a non-profit organization's travel costs by nearly \$5,000. However, if we measure operations research by its ability to "do good things" and the degree to which model results are implemented in actual practice, then our approach may be beneficial.

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Figure 1
Map of Parish Locations



Table 1
List of Pastors and Stewardship Parishes

Pastor (** = resides outside of Saskatoon)	Stewardship Parish
Gerry **	Saskatoon 1 st
Kim	Saskatoon 2 nd
Larry **	Saskatoon 3 rd
Steve L **	Saskatoon 4 th
Alan	Flin Flon
Hans	Melfort
Richard	Prince Albert
Steve W	North Battleford
John	Meadow Lake
Stan	Kindersley
Armes	None

