



Curling's paradox

Kent J. Kostuk^{a,*}, Keith A. Willoughby^b

^a*Federated Co-operatives Limited, 401-22nd Street East, Saskatoon, Canada SK S7K 3M9*

^b*Department of Management, Bucknell University, Lewisburg PA 17837, USA*

Available online 25 November 2004

Abstract

There is an age old debate in curling about whether it is better to be down one point with last shot, or ahead by one point without. The objective of this paper is to apply sensitivity analysis as a methodology to differentiate what appears to be two seemingly equal scenarios. A probability tree is developed for each scenario and a comparison is made based on an expected value basis. Sensitivity analysis is performed to determine whether preference changes with changes in the key parameters. Indeed, preference is impacted by changes in these parameters. In general there is no universally preferred scenario; under specific conditions a general preference can be established. Ultimately, preference for one scenario over the other is based on an individual's perception of the probability of scoring with last shot.

© 2004 Published by Elsevier Ltd.

1. Introduction

As with a variety of other sports (most notably basketball, football, or baseball), owning the final shot, possession or at-bat offers certain strategic advantages. The sport of curling is no different. Having the final shot (the so-called "having the hammer") may permit a team some degree of confidence. Should their final stone be a successful shot, they could reap the reward of scoring a multiple number of points. In the strategic instruction provided by coaches to younger players, significant emphasis is given to the critical role played by last shot. An age old adage "score two with the hammer, give up one without" is frequently repeated.

* Corresponding author. Fax: +1 306 373 8255.

E-mail addresses: k.kostuk@fcl.ca (K.J. Kostuk), kwilloug@bucknell.edu (K.A. Willoughby).

In curling there is an age old debate on whether it is better to be ahead by one point without last shot, or down one point and have last shot. The “score two with the hammer” mantra rings strong in most curlers’ minds. As in all sports, what *should* happen and what *does* happen are two different things. Historically, the scenarios were differentiated based on the curlers’ confidence (and implied subjective probabilities) in their ability to score two or more points when they had last shot during a game’s final stages.

It should be noted that in practise one would rarely, if ever, actually have to make a strategic choice between being one up without the hammer, or to be down one with. Anecdotally, based on over 2000 games played by one of the authors (several hundred at an elite level of competition) the scenario has only presented itself once (see the appendix for a detailed description). In spite of its infrequent occurrence, the scenario is a hotly debated topic within the curling community. This is in large part, because in a closely contested game between two well matched teams, the teams’ philosophical preference will influence their general strategy (particularly in the last half of the game) in an attempt to achieve their preferred scenario for the last end. If one of these scenarios could be identified as being better than the other, it would effect how the game is played and taught.

In previous research by Willoughby and Kostuk [1] a group of more than 100 competitive curlers (including several Canadian and world champions) were surveyed. As expected there was no consensus on a preferred scenario. The analytical model developed in that paper did not provide a conclusive solution, but it did determine that preference was dictated by the respondents’ perceived ability to score points throughout the game.

The objective of this paper is to apply sensitivity analysis as a methodology to differentiate these seemingly equal scenarios. Failing that, the next best option is to determine the circumstances under which each scenario is preferred. The layout of the remaining sections of this paper is as follows. The second section of the paper provides a background on the sport of curling and an introduction to decision analysis. In this section, basic concepts and vernacular will be introduced. The third section of the paper develops the analytical model used to compare the two scenarios and outlines the general research methodology. The analytical model is applied in section four. The final section summarizes and discusses the results.

2. Background

Curling is a winter sport with European roots; a 16th century painting by Flemish artist Peter Bruegel provides evidence that the sport was played on continental Europe during that period. During the 17th century, a formalized set of rules were developed in Scotland. The first North American curling club was established in Montreal, Quebec, in 1807; the Orchard Lake Curling Club, established near Detroit, Michigan in 1832 was the first club based in the United States [2].

The rules of curling are quite basic. Two teams, made up of four players each compete on an ice surface 14 feet wide and 146 feet long. The ice surface is known as a sheet, and at each end of the sheet are concentric circles with diameters of 12, eight, and four feet. Collectively these circles are known as the rings or the house. The objective of the sport is to propel 45 pound disks of granite (stones) as close to the middle of the rings as possible. Teams take alternate turns, with each member of the team delivering two stones (one at a time) from one end of the sheet to the other. Teams have the choice of putting their stones in the rings, removing their opponent’s stones from play with their own stones, or, delivering their stones in front of the rings to guard their stone(s) in the house. Once all the stones are delivered, the score

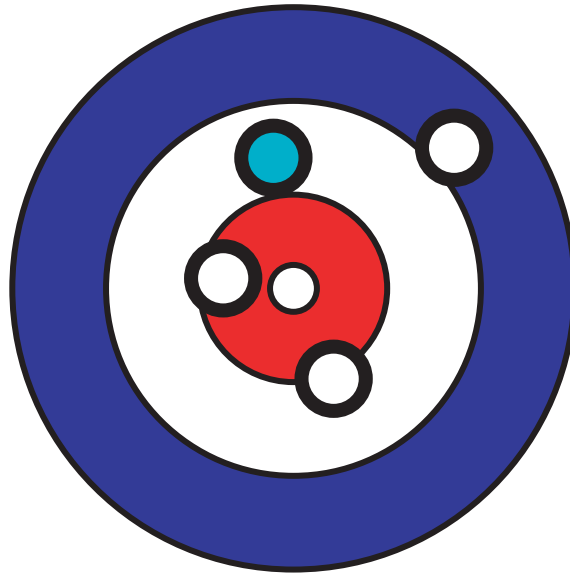


Fig. 1. An example of a team scoring two points.

is tallied for the “end.” The team with the stone(s) closest to the centre of the rings, count one point for every stone that is closer to the middle of the house than any of their opponent’s stone(s). Fig. 1 illustrates an arrangement of rocks where one team (the white rocks) scores two points.

As stated earlier, the team that throws last has “the hammer.” In the beginning of the game, the hammer is determined by a coin toss. In subsequent ends the hammer is determined based on scoring; the team that is scored upon gets the hammer in the following end. If no scoring occurs the team retains the hammer. If a team is faced with an empty house with their last rock, they have two options: attempt to throw their stone into the rings for one point (sacrificing the hammer in the following end), or intentionally missing the rings (usually by throwing through the rings) so they retain the hammer in the following end. If the team without the hammer scores a point that is known as a steal (e.g. a team without the hammer scoring three points in an end is said to have stolen three points). A standard game lasts 10 ends. If the score is tied after 10 ends, play continues until one of the teams scores. The likelihood of a game extending past 11 ends is rare, and past 12 is negligible. In the 902 games played at the Canadian Men’s Curling Championship (the Brier) between 1985 and 1997, only four games took 12 ends to decide the winner.

Decision analysis is an approach used to determine the optimal choice when faced with a set of alternatives in the face of uncertainty. There are numerous decision analysis applications in the domains of business and sports: the decision to seed hurricanes to reduce their destructive force [3]; managing R&D [4]; and, evaluation of a student athlete drug-testing program [5].

Mathematically, decision analysis is very straight forward. The first step is determine the decision. Next, a set of alternatives are generated. For each alternative a set of events must be identified. Probabilities for the outcomes of each event are then determined. Analysis is usually completed on an expected value basis; the value of each event, multiplied by the appropriate probabilities generates a metric on which a decision can be evaluated.

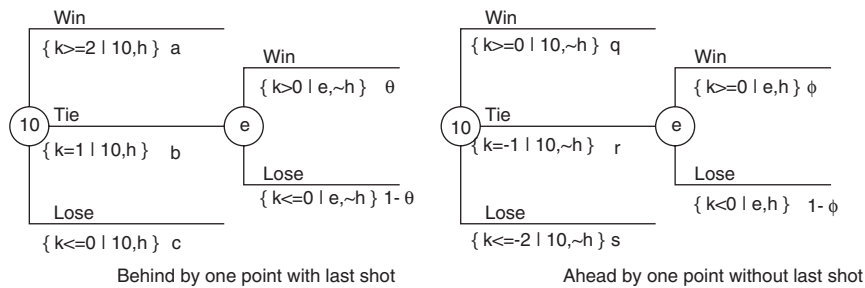


Fig. 2. Probability trees for when a team is behind with the hammer and ahead without the hammer.

3. Model development

This paper investigates whether it is better to be up one point without the hammer, or one down with. Under each scenario the differential is a single point. This suggests (at least for this game) the teams are of comparative ability. As previously noted, the likelihood of encountering the decision to choose between these scenarios in a game is extremely small. In fact, this investigation is more of an abstract comparison of two potential events. But in practise, a team's preference for one scenario over the other can influence a team's strategy for the latter half of the game.

In either scenario there are only three possible outcomes: Win, Tie, or Lose. If there is a tie in the tenth end, the teams must play an extra end for which the potential outcomes are either Win or Lose.

It would appear from Fig. 2 that only one extra end can be played in a game. This is not the case, but in practise, the likelihood of playing two extra ends is so low that in effect the probability is 0. To simplify the analysis, only two outcomes were considered for the extra end: winning (W) and losing (L).

When the team has the hammer in the tenth end, the probability of winning is equal to the probability of scoring two or more points in the tenth end ($a = \{k \geq 2 \mid 10, h\}$) plus the probability of a tie in the tenth end multiplied by the probability of winning in the extra end without the hammer ($b\theta = \{k = 1 \mid 10, h\} \{k > 0 \mid e, \sim h\}$). Similarly, when the team does not have the hammer, the probability of winning is equal to the probability of blanking or scoring any points in the tenth end ($q = \{k \geq 0 \mid 10, \sim h\}$) plus the probability of a tie in the tenth end multiplied by the probability of winning in the extra end with the hammer ($r\phi = \{k = -1 \mid 10, \sim h\} \{k > 0 \mid 10, h\}$). By comparing the probability of winning when down one point in the tenth end with the hammer ($\{W \mid 10, h\} = a + b\theta$) with the probability of winning when up one point in the tenth end without the hammer ($\{W \mid 10, \sim h\} = q + r\phi$) we can determine a preferred scenario.

Winning in the extra end with the hammer, describes the same outcome as losing in the extra end without the hammer. From a normative decision making perspective, the probability of winning in the extra end with the hammer (θ) should be equal to the probability of losing in the extra end without the hammer ($1 - \phi$); thus, $\theta + \phi = 1$. In the authors' previous work, this relationship was not presumed [1]. In that work, 53 of 113 respondents of a survey of elite curlers (including several past world champions) provided separate values for θ and ϕ which summed to unity; 69 of 113 respondents provided values that summed between 1 and 1.1. Though hardly conclusive, this does suggest that even in a subjective environment, the majority of curlers would support a classical view of these probabilities.

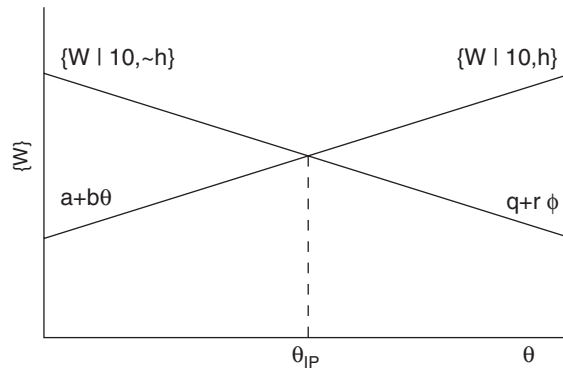


Fig. 3. A graphical interpretation of the indifference point.

A similar argument can be made for stating that the probability of:

- tying the game in the tenth end with and without the hammer are equivalent probabilities ($b = r$);
- and, the probability of scoring two points and giving up two points with and without the hammer are equivalent ($a = s$).

These new relationships help define $\{W | 10, \sim h\}$ in terms of the variables defined by $\{W | 10, h\}$: $\phi = 1 - \theta$, $r = b$, and $q = 1 - (a + b)$. Consequently, $\{W | 10, \sim h\} = 1 - (a + b\theta)$, which allows us to determine under what conditions the two are equal ($\{W | 10, h\} = \{W | 10, \sim h\}$). Fig. 3 is a graphical interpretation of equating these two scenarios. The intersection point represents a point of indifference; for this value of θ one would be indifferent to either scenario. The indifference point can be determined algebraically to be

$$\theta_{IP} = \frac{(1 - 2a)}{2b}. \tag{1}$$

Thus, for any combination of a and b one can determine an indifference point (θ_{IP}). The significance of the indifference point is that for $\theta < \theta_{IP}$ the preferred scenario is to be ahead by one point without the hammer. The converse is true if $\theta > \theta_{IP}$.

4. Analysis

In the previous section, three variables were identified as key to the decision making process: a , b , and θ . Plotting θ as a function of a and b allows us to illustrate the problem space (Fig. 4). The problem space can be constrained based on some simple observations.

When a team has the hammer, the probabilities of the three possible events must sum to 1 ($a + b + c = 1$). Since each of these probabilities must be non-zero and each must be less than or equal to 1 ($0 \leq a, b, c \leq 1$) we know that $a + b \leq 1$.

Fig. 3 demonstrated the relationship between θ and θ_{IP} . For the situation where $\theta_{IP} = 0$ we know that for all values of θ , $\theta \geq \theta_{IP}$, which in turn means that under all circumstances one down with the hammer in the tenth end is preferred. This constraint is determined by solving Eq. (1) for $\theta_{IP} = 0$: $a = 0.5$. Given

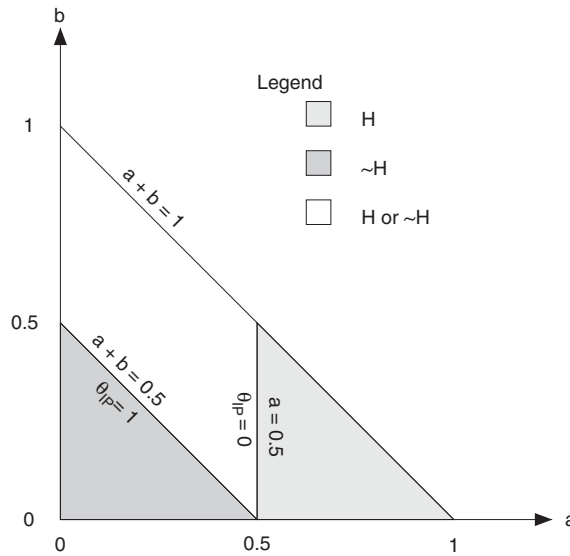


Fig. 4. Constraining the problem space.

the complementary relationship between $\{W | 10, h\}$ and $\{W | 10, \sim h\}$; if $a \geq 0.5$ then $\{W | 10, h\} \geq 0.5$ which means that $\{W | 10, h\} \geq \{W | 10, \sim h\}$.

Conversely, when $\theta_{IP} = 1$, for all values of θ , $\theta \leq \theta_{IP}$. The preferred scenario is to be ahead without the hammer. When $\theta_{IP} = 1$: $a + b = 0.5$. If $a + b < 0.5$, then the probability of scoring any points in the tenth end with the hammer is less than 0.5 which implies that not having the hammer is better than having the hammer.

The mathematical constraints imposed on the solution space define three distinct regions in Fig. 4:

- if $a \geq 0.5$, then one would always prefer to be one down with the hammer in the tenth end,
- if $a + b \leq 0.5$, then one would always prefer to be one up without the hammer in the tenth end,
- and, if $a \leq 0.5$ and $a + b > 0.5$, then one’s preference is based on their perception of a , b , and θ .

A practical constraint that could be imposed on this space is based on our knowledge of θ . Obviously, a team prefers having last shot over the alternative since scores are tallied immediately after the final rock of an end. This suggests that $\theta < 0.5$. Plotting $\theta_{IP} = 0.5$ expands the area under which one would prefer to be one up without the hammer (Fig. 5). This new region is bounded by $\theta_{IP} = 1$ on the left and $\theta_{IP} = 0$ on the right; in between are infinite combinations of a and b , all generating θ_{IP} values between 0.5 and 1. From a practical perspective, under all cases, $\theta_{IP} > \theta$. The subsequent impact is that now:

- if $a \geq 0.5$, then one would always prefer to be one down with the hammer in the tenth end,
- if $2a + b < 1$, then one would always prefer to be one up without the hammer in the tenth end,
- and, if $a \leq 0.5$ and $2a + b > 1$, then one’s preference is based on their perception of a , b , and θ .

The solution space can be further reduced if constraints based on empirical or subjective data for a and b are applied. Based on statistics from the Brier (1985–1997) there were 221 games which entered the tenth

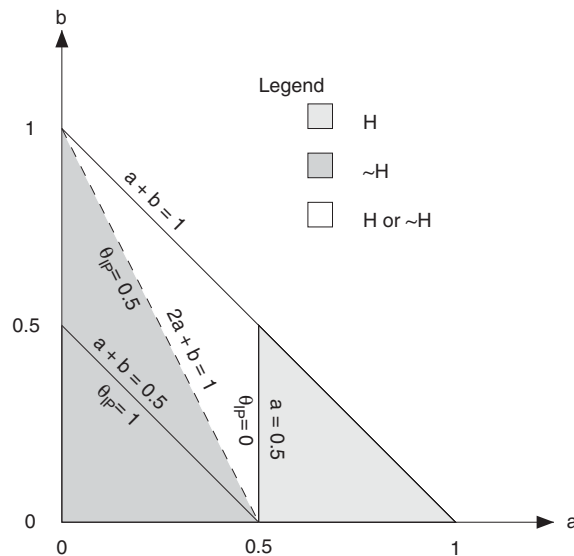


Fig. 5. Imposing further limitations on the solution space based on $\theta < 0.5$.

end with a one point difference. Of these games, 61 resulted in a blank or the team without the hammer scoring; in other words $c = \{k \leq 0 \mid 10, h\} = 0.276$. In the survey of 113 competitive curlers in Willoughby and Kostuk [1], 94% felt that $c \leq 0.25$, and 98% felt that $c \leq 0.30$. The similarity of both the empirically and subjectively derived probabilities suggest it is reasonable to state that $a + b \leq 0.75$. There is also a practical limitation on b (the probability of scoring a single point in the tenth end with the hammer). The Brier statistics suggest that $b = 113/221 = 0.511$ and 101 of the 113 curlers felt that $b \leq 0.5$.

A practical upper bound on a can also be imposed on this space. Brier statistics suggest $a = 47/221 = 0.213$. Unlike b and c , the survey results for a did not correlate with the Brier data. The competitive curlers perceived they had a much better chance of scoring two or more points in the tenth end; 84 of 113 curlers felt that $a \leq 0.5$, 93 of 113 felt that $a \leq 0.6$, and 112 of 113 felt that $a \leq 0.7$. If the practical limit for $a = 0.25$, then there is no practical combination of a , b , and θ where one would prefer to be one down with the hammer. The large variation between the perceived probability and the observed data suggests that a 's value is a significant contributor to the controversy for which scenario is preferred.

Fig. 6 illustrates the impact of imposing the constraints: $a + b \leq 0.75$ and $b \leq 0.5$. The key impacts are that the size of the solution space has been reduced and the relative weighting of the preferred scenarios has changed. From a decision making process, there is no impact.

5. Conclusions

The ultimate objective of this research was to determine if there was a definitive solution to the age old question: is it better to be one down with the hammer, or one up without? Based on the work presented here, it is safe to say that there is no definitive solution. A fundamental assumption was made that the probability distributions associated with scoring with and without the hammer are symmetric.

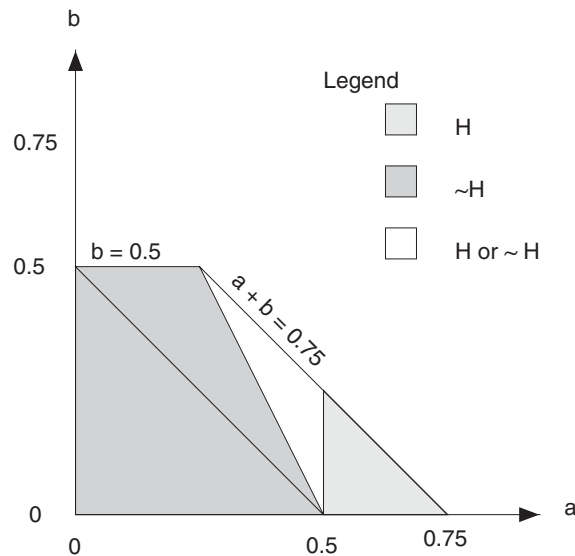


Fig. 6. Imposing empirically and subjectively derived constraints on the solution space.

This resulted in the development of a simple two variable equation that could be used to indicate the indifference point between the two scenarios. By imposing mathematical and practical constraints on the solution space, the conditions under which either scenario would be preferred were illustrated.

Under specific conditions the following statements can be made. When the probability of scoring more than one point in an end (with the hammer) is greater than or equal to 0.5 the preferred scenario is to be down one point with the hammer. If the probability of scoring one or more points with the hammer is less than 0.5 then under most reasonable circumstances, the preferred scenario is to be ahead one point without the hammer. If statistics from the Brier are used as a practical limitation on the probability of scoring more than one point in the tenth end, then the preferred scenario is to be up one without the hammer.

Preferring to be down one point with last shot is mathematically tractable, but under practical situations, it is safe to say that being ahead one point with the hammer is a more general rule of thumb.

Appendix

As stated earlier, in over 2000 games played, one of the authors has only once been involved in a game where one team was faced the decision to enter the tenth end one up without the hammer, or one down with. The game was a semi-final in a World Curling Tour event. Team A were the defending champions, and team B, 2 years removed from being both Canadian and world champions. The ice conditions were such that the game was low scoring. After eight ends the former world champions were ahead 1–0. As the ninth end progressed it appeared that the best alternative for team A was to play for a blank end and to be one down with the hammer (the team's preferred strategy). With four rocks left to play, and team A's third last shot sitting on the button (center of the rings) team B missed with their second last rock. Team

A was then in the situation where it had the choice of protecting the rock in the rings with another rock (a guard), expecting to score two, and thus be ahead by one going into the tenth end, but without last rock. Alternatively, the team could remove its own rock from the rings to ensure there would be no score in the end and thus retain the hammer and be one point down. Team A elected to remove its own rock from the rings (in spite of the author's advice), and then successfully blanked the end. Team A scored a single point in the tenth end. The game took 12 ends to complete (another unique twist to the story), with team B ultimately winning the game.

References

- [1] Willoughby KA, Kostuk KJ. Preferred scenarios in the sport of curling. *Interfaces* 2004;34(2):117–22.
- [2] Murray WH. *The curling companion*. Toronto, Canada: Collins; 1981.
- [3] Howard RA, Matheson JE, North DW. The decision to seed hurricanes. *Science* 1972;1191–202.
- [4] Stonebraker JS. How Bayer makes decisions to develop new drugs. *Interfaces* 2002;32(6):77–90.
- [5] Feinstein CD. Deciding whether to test student athletes for drug use. *Interfaces* 1990;20(3):80–7.