The Disposal of Excess Stock:

A Classification of Literature and Some Directions for Further Research

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Abstract:

This paper presents a survey of the various approaches used to determine the optimal inventory disposal amount, given a situation in which an organization finds itself with an excess of stock on hand. We examine both simple decision rules and analytical models that have been developed to deal with this problem. Analytical models are categorized into ones that consider the disposal decision in isolation, and those models in which an acquisition decision is combined with a concomitant disposal choice. We present some key directions for further research in this inventory management area.

Key words:

Excess stock disposal, classification system

1. INTRODUCTION

A critical inventory management decision arises when an organization finds itself with an excess of stock on hand. Specifically, the problem is to determine the appropriate amount of stock to dispose. Disposal creates benefits in at least two ways; namely, the salvage revenue obtained from surplus unit disposal, and the savings in inventory carrying charges since less stock is now held. However, due to ongoing operational usage of this item, the organization may be required to eventually repurchase units from its supplier. Eliminating "too much" of this stock may, thus, force the company to make premature repurchasing arrangements. As a result, the cost tradeoff exists between salvage revenue and reduced inventory carrying charges versus future repurchasing costs.

The potential causes of excess stock are legion. An abrupt decrease in demand or changing business conditions may lead to an excess stock situation. Similarly, price increases, forecasting errors, customer cancellations, the introduction of a new (competing) product, production overruns, overpurchasing (to protect against stockouts), or even simple goofs (e.g. errors in the transmission of an order request) may be the basis for the excess occurrence. Poor quality in final product assembly could lead to an over-supply of a sub-component. Ultimately, inadequate materials planning and execution systems are central to the problem of excess stocks.

From an equipment replacement context, excess stock may arise due to the availability of better and cheaper equipment. The older equipment, consequently, becomes "excess" in comparison with the current requirements of the organization. Technical obsolescence or physical deterioration may also render older equipment excess and useless.

Excess stock can result in a project management environment due to engineering design changes (some of these design changes occur after the initial procurement of materials has been made). The inherent uncertainty in specific types of projects (e.g. subsurface work such as pipeline construction) may also lead to an excess of stock on hand.

Tersine and Toelle [37] suggest that excess inventory is a "dead weight". Among other adverse effects, it uses valuable storage space, inflates assets, diminishes working capital, and causes a reduction in return on investment (ROI). Toelle and Tersine [39] claim that inventory is in fact a liability if it costs more than it earns. They suggest a variety of means of disposing of excess stock: return to supplier, third-party sale, and even charitable donation. Gottlieb [13] submits that two-thirds of the U.S. national defense stockpile is wholly or in part excess. This surplus stock represents an investment of a few billion dollars. He further alludes to the political difficulties and economic disruptions that can be created should a country be perceived as "dumping" excessive amounts of key materials. Bolwijn and Kumpe [4] cite Mr. Martin Kuilman (a Philips Vice-President), who maintains that the company has a substantial investment tied up in unnecessary inventories of subassemblies, finished products and raw materials.

The current authors recall the story of a firm that contracted with a large construction company to purchase all of the latter's excess pipeline during a given year. The firm received such a quantity of surplus pipeline that it quickly filled its own warehouse facilities. It was eventually required to rent storage space in order to house the excess stock!

The surplus stock disposal function comprises an integral role in any effective materials management system. Bennett [3] defines materials management as coordinating the various operations of purchasing, inventory control, warehousing, distribution and disposal of surplus materials. Kathawala and Nauo [19] claim that integrated materials management ought to be viewed from a holistic perspective. The actions of planning, acquisition, control and disposal should be performed in such a way that facilities, personnel and capital are optimized, while providing appropriate customer service levels. The authors remark that the disposal function, once regarded as an incidental task, has gained substantial importance due to a better recognition of the key benefits it can generate in an organization. Silver, Pyke and Peterson [33] report that, given the current increases in the rate of technological change (which imply a shortening of the typical product life cycle), the general area of excess stock disposal is likely to continue its increase in importance.

A considerable number of researchers have examined the excess stock disposal problem. This paper presents, in our view, an initial attempt to categorize the various approaches. The format of the paper is as follows. Section 2 describes simple decision rules for the disposal of excess stock. Section 3 relates various analytical disposal models. This section is divided into those models which consider the disposal decision in isolation ("strict" disposal), and those ones in which the disposal choice is combined with an acquisition decision ("hybrid"). The "strict" disposal models are further subdivided into those that examine deterministic or stochastic usage. The "hybrid" area is split into general acquisition and disposal models, and models that examine quantity discounts and disposal. Figure 1 illustrates the framework adopted for this paper. This survey paper concludes with some key directions for further research.

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2. SIMPLE DECISION RULES

These decision rules can be viewed as simple (mainly subjective) "rules of thumb". They pay little (or no) attention to such details as inventory carrying charges and future reordering costs. Their value is not so much in "to-the-penny" exactitude, but rather in their ability to offer managers a conceptually simple, easy to implement technique of determining the amount of excess stock an organization ought to dispose.

Pattinson [27] prescribed a cross-functional approach to this problem, involving representatives from such departments as marketing, operations, finance and engineering. He suggested that one closely monitor all inventory in excess of 12 months' supply. Any stock exceeding that time supply would be considered surplus to current requirements.

Brown [5] offered a description of the excess stock issue for a general managerial audience. He advocated using managerial intuition in setting two limits, the "number-of-weeks supply" and the "dollar-value-of supply". Any stock that surpassed either of these two limits would be regarded as excess inventory.

A more quantitative, yet still relatively simple, excess stock rule was given by Silver, Pyke and Peterson [33]. They suggested that one calculate, on an item-by-item basis, the expected time at which the inventory level would be depleted. This quantity was known as the item's "coverage". It is represented as:

$$CO = \frac{12 * I}{D}$$

where:

CO = coverage, in months
I = on-hand inventory, in units
D = expected usage rate, in units per year

By listing each item in descending order of coverage and also maintaining a record of the item's unit value, managers could obtain a quick indication of the "cost" of excess stock. For instance, they could easily determine the percentage of total inventory value tied up in stock with coverage of at least, say, 30 months. This would provide decision-makers with evidence of the significance of the excess stock problem. Disposal of a portion of the inventories of items with at least a certain coverage would "free up" a specific amount of inventory investment.

3. ANALYTICAL MODELS

Although simple decision rules can provide a quick basis for decisions regarding the disposal of excess stock, analytical models can consider a variety of specific inventory details. The outcome of these modelling efforts is the quantity (and in the case of some models, the "timing") of excess stock disposal.

3.1 "Strict" Disposal Models

Several models have been developed to examine the disposal of excess stock, given that the organization is currently in a surplus inventory situation. We refer to these as "strict" disposal models. Researchers have attempted to determine either an economic retention quantity (or economic retention time period). Any stock that is found exceeding either the best retention quantity or time supply ought to be disposed. Marginal salvage values for stock disposal have been assumed to be constant in all cases. However, these models have analyzed the problem from a few different angles. Besides the use of different demand distributions (which are illustrated in the subsequent sections), models have also differed in the various cost components used, and the manner in which inflation and the time value of money are addressed.

3.1.1 Deterministic Usage

These types of models have assumed that future demand for the item is both known and constant. Simpson [34] was an early contributor to the excess stock problem. Basing his analysis on inventories held at Naval supply stores, he calculated an economic retention time period. His break-even examination featured a tradeoff between storage and obsolescence costs versus the expenses of repurchasing the material in the future (if and when required). The author used a constant probability of obsolescence, and ignored inflation (ie. the future unit acquisition cost was assumed to be equivalent to the current price). Simpson's economic retention period was:

$$\frac{\ln\left[\dfrac{D(i+p)+r(1-p)\ (1+i)}{i+p+r(1-p)\ (1+i)}\right]}{\ln\left[\dfrac{1-p}{1+i}\right]}$$

where:

D = fraction of unit acquisition cost obtained when item is disposed
i = interest rate
p = probability of obsolescence
r = storage cost of item (expressed as a fraction of the item's dollar value)

The value resulting from this expression, when multiplied by the constant annual demand, would give the economic retention quantity.

Mohan and Garg [23] expanded some features of Simpson's earlier model. Besides considering inflation, they allowed for the obsolescence factor to follow a general probability distribution. In fact, they used the exponential distribution and constructed an appropriate economic retention period. The effect of using this distribution (over Simpson's constant probability of obsolescence) would be to suggest a higher obsolescence likelihood during the early stages of the planning horizon. As a result, economic retention periods would diminish. Kulshrestha [21] expanded Simpson's model by incorporating an exponential probability distribution during initial deterioration and obsolescence, up until the time at which one could model the obsolescence as following a normal distribution.

Naddor [24] developed an excess stock disposal model for the cases of both finite and infinite horizons. However, he did not include any present value considerations in his analysis. Dave and Pandya [9] expanded Naddor's model by allowing the stock to exhibit a constant rate of deterioration. They examined a classical lot-size inventory system, in which the economic order quantity (EOQ) was used for future, ongoing replenishments. Assuming no shortages and zero leadtime, they developed expressions for the best amount of surplus stock to retain. Under no conditions would an organization dispose a quantity of such a size as to leave themselves with less than the EOQ on hand. Dave [8] elaborated on the previous work, by developing models in which shortages were permitted to be completely backlogged.

Hart [15] recognized that demand rates may be variable during the planning horizon. However, he assumed that this horizon could be divided into a given number of subperiods (which would not necessarily be of the same length), and that a separate forecast of demand could be generated for each of these respective time periods. The demand rate, then, was presumed to be constant within each of these subperiods. In this way, item usage was deterministic, yet time-varying. Hart heuristically determined a future procurement schedule for the item, and noted that the heuristic performed quite well when compared to the optimal schedules produced by a dynamic programming algorithm (this latter technique required considerably longer computing times, a major consideration when thousands of inventory items would be examined). Specific costs considered included inventory holding charges, fixed and variable procurement costs, and scrap value of disposed units. After discounting all future costs to the present, he was able to find the optimal retention quantity by using a sequential search procedure.

An additional effort to recognize deterministic, time-varying demand was produced by Miller, Mellichamp and Henry [22]. Basing their research on a troubled General Motors carburetor assembly plant in Tuscaloosa, Alabama, they attempted to find minimum cost time supplies for surplus items. Due to the multiproduct nature of the facility, the same product could go into several "kits". As a result, there existed an additional manner in which excess stock could be disposed. Surplus units could be "remade" into a different product for which there was a "good" demand. Their present value model considered inventory carrying charges as well as future procurement costs. Since future replenishments of the item would most likely be produced on a smaller lot-size production run, the unit acquisition cost was assumed to be higher in the future. Other costs considered included salvage revenue and the tax savings associated with inventory write-offs. Using the technique of differencing, the authors determined the integer value of time supply that yielded the smallest total discounted cost. Adoption of the analytical method generated savings of approximately $1 million at the GM plant.

Krupp [20] illustrated the manner in which obsolescence can lead to excess inventory. He defined "fiscal obsolescence" as the gradual depletion in a product's value, resulting from the effect of accrued carrying charges over an extended period of time. At the specific point in time at which the cumulative carrying charges exceeded the net unrecoverable value of the item (standard cost less resale or salvage value), the product ought to be considered obsolete (and hence, excess). Any stock which exceeded this economic time supply would be disposed. However, he neglected to include such factors as time value considerations, or the effect of future reordering and repurchasing costs.

Measuring alternative disposal strategies in terms of their effects on relevant cash flows, O'Neil and Fahling [25] presented a decision model for excess inventories. Their cash flow liquidation model, possibly appropriate for a retailing or distribution enterprise, evaluated the present values of inventory carrying charges and cash from disposal (net of tax). However, they did not incorporate future reordering and repurchasing costs. They allowed disposal of stock at "discrete" points in time (the end of each month). A rather cumbersome procedure was developed to determine the best disposal strategy. The authors evaluated the total discounted cash flow of retaining all inventory, then the total discounted cash flow of liquidating one month of inventory (and retaining the remainder), and so on until all possible liquidation quantities up to and including the quantity on hand had been evaluated. The optimal disposal amount, then, was the one leading to the maximum discounted cash flow. Tersine, Toelle and Schwarzkopf [38] expanded O'Neil and Fahling's liquidation model. They adopted continuous compounding of future cash flows (the earlier authors had suggested that all cash flows occurred at the end of discrete time periods). In addition, the later researchers were able to develop an analytical, closed-form result for the optimal number of months of stock to retain.

Brown [6] determined economic retention quantities by comparing current salvage revenue with the future costs of repurchasing the item. He permitted the consideration of the time value of money, but disregarded holding costs and the fixed cost of placing a future replenishment order.

A managerial examination of the nature of excess stocks was provided by Doll [10]. He proposed the Inventory Evaluation and Review Technique (INVERT), a process for reviewing the present state of an organization's inventory position and providing guidelines for improvement plans. While he suggested that an economic analysis be performed to determine the most beneficial disposal strategy (retain, sell, segregate, write-off), he failed to indicate any analytical details of this procedure.

Tersine and Toelle [37] generated relationships for the economic time supply of an item, under the existence or non-existence of present value and inflation considerations. Backorders were not allowed. Their "net benefit" for the disposal of excess stock may be conceptualized as:

Net Benefit = Salvage Revenue + Holding Cost Savings - Repurchase Costs - Reorder Costs

When present value and inflation were neglected, the following result was obtained for the economic time supply:

$$\frac{P - P_s + \frac{C}{Q}}{PF} + \frac{Q}{2R}$$

where:

P = unit acquisition price
P_s = salvage value
C = ordering cost
Q = item's lot size
F = holding cost fraction
R = annual item demand

Under the scenario of present value considerations and adjustment for inflation, the economic time supply became the value of t (t_o) that satisfies:

$$\left[\frac{PFR}{2k} - \frac{PFtR}{2}\right]e^{-kt} + \left[\frac{PFQ}{2} + \frac{PQ(i-k) + C\ \ (i-k)}{e^{(i-k)Q/R}\ -\ 1}\right]e^{(i-k)t} - P_s R - \frac{PFR}{2k} = 0$$

where:

i = expected inflation rate
k = required rate of return

Tersine and Toelle were unable to solve the above relationship analytically for the economic time supply, t_o. As a result, they were required to use Newton's method to find a numerical solution. It should come as no surprise that the values for t_o given by the authors' two models were different. The economic time supply produced by the present value model was lower than the former one since reorder and repurchase charges, incurred in the future, can be heavily discounted when considering present values. This would tend to reduce the appeal of retaining more units of excess stock. As an outgrowth of their models, Tersine and Toelle computed the minimum economic salvage value, the lowest price for which a unit of excess stock would be disposed. This has considerable managerial appeal, since it provides some indication of the sensitivity of solutions to changes in model parameters.

Silver, Pyke and Peterson [33] described a method to calculate the amount of excess stock which ought to be disposed. Neglecting any present value considerations, they determined the optimal disposal quantity as:

$$I - EOQ - \frac{D(v - g)}{vr}$$

where:

I = current inventory level of the item
EOQ = replenishment lot size (traditional economic order quantity)
D = annual item usage
v = unit acquisition cost
r = inventory carrying charge (expressed in \$ per \$ of inventory per year)

The authors note that, when $v = g$ (ie. the salvage value is equivalent to the unit acquisition cost), the best disposal strategy is to dispose down to the EOQ level. In this case, it is optimal to put the inventory into the same situation as immediately subsequent to the receipt of a replenishment.

George [12] determined the minimum disposal price one must receive to obtain immediate disposal of an entire stock of surplus units. Restricting attention to a slow-moving, non-replenishable item, he assessed different pricing strategies by comparing the net return on invested funds. He found that, in most practical situations, a special price of at least 80% of the normal price would be required for entire surplus disposal.

Stulman [35], ignoring the possibility of obsolescence or spoilage, developed an expression for the optimal retention quantity. His net benefit of excess stock disposal may be characterized as:

Net benefit = Immediate scrap revenue - Present value of attrition-period holding costs
- Present value of all post-attrition-period regular operating costs

The "attrition period" consisted of that time period, subsequent to the disposal decision, in which remaining items were "depleted" by ordinary usage. Since no stock needed to be ordered during this time, the only relevant costs involved holding charges. Stulman assumed that this attrition period concluded when the remaining inventory reached the top of its normal operating range. Given the difficulty of clearly defining some of the above terms, he developed an approximate procedure in finding the optimal retention amount. The best such quantity was the value of y that minimizes:

$$c(x - y) + \frac{hs\ e^{-i(y-s)/a}}{i} - \frac{ha\ e^{i(y-s)/a}}{i^2} - \frac{hy}{i} + \frac{ha}{i^2} - F\ e^{-i(y-s)/a}$$

where:

c = unit salvage value
x = inventory currently on hand
y = number of units retained
h = holding cost per item per unit time
s = maximum inventory under regular operations
i = annual discount rate

a = annual demand
F = present value of all future operating costs of regular inventory operations

Despite the fact that the above expression could be differentiated with respect to y, Stulman found that it was rather difficult to solve for the best quantity. Consequently, he used a search technique to find the value of y that gave the economic retention quantity.

3.1.2 Stochastic Usage

Rosenfield [29] was the initial researcher to study the problem of excess stock disposal, given stochastic demand. The number of units demanded per "demand episode" was assumed to follow a Poisson distribution. While he assumed no stockouts and no additions to inventory, the author did consider such factors as the immediate salvage value of surplus unit disposal, holding costs resulting from carried items, and the ultimate sales value of surplus stock. He applied his methodology to an actual distributor of durable goods faced with excessive amounts of slow-moving items. The model showed that substantial savings could be earned by the judicious disposal of surplus stock. Rosenfield also examined the effect of inventory perishability on the excess stock disposal decision. He obtained closed-form results for the cases of complete (all-units) perishability at random or known times, and for individual item perishability at random times.

In a later paper, the above author [30] showed the optimality of a myopic policy when disposing excess stock. Assuming that the same disposal opportunity is available at any subsequent time and given Poisson demand, the optimum threshold remains the same. Notwithstanding the opportunities to change one's mind in the future, one still disposes the same number of items. The retention decision, then, can be made without any examination of future decisions. The myopic policy gives the economic retention quantity as the largest integer less than:

$$\frac{\ln\left[\dfrac{V+\dfrac{r}{i}}{A+\dfrac{r}{i}}\right]}{\ln\left[\dfrac{\lambda}{\lambda+i}\right]}$$

where:

V = salvage value as a percentage of current value
r = storage costs per unit time as a percentage of current value
i = discount rate
A = average ultimate sales value as a percentage of current value
λ = average number of units demanded of an item per unit time

Continuing the analysis done with deterministic demand, Stulman [35] found an expression for the optimal retention quantity given probabilistic (Poisson) usage. The best amount to retain was the largest integer quantity less than:

$$\frac{\ln\left[\dfrac{c+\dfrac{h}{i}}{\dfrac{Fi}{\lambda+i}+\dfrac{h}{i}-\dfrac{h(s+i)}{\lambda+i}}\right]}{\ln\left[\dfrac{\lambda}{\lambda+i}\right]}+s+1$$

where:

λ =Poisson demand rate, and all other terms are as described previously

Hill, Giard and Mabert [16] analyzed service parts inventory retention levels in a Fortune 100 company. They developed an integrated, menu-driven, databased decision support system (DSS) that permitted the forecasting of future demand and the determination of optimal retention stocks. The authors permitted disposal to occur immediately, or at the "product termination date". The latter time was established by marketing as the period after which no parts required for the specific product were to be kept in stock. Various components such as both tax savings and after-tax revenue of surplus unit disposal (either immediately, or at the termination date) and after-tax back-ordering and carrying costs were included in their model. Essentially, the retention stock was chosen by the following equation:

$$E(Y_T) + zV(Y_T)^{1/2}$$

where:

$E(Y_T)$ = expected life time demand for the service part determined from the forecasting model

$V(Y_T)$ = variance of life time demand for the service part determined from the forecasting model

z = safety factor

The decision variable, then, became the appropriate value of z that minimized total relevant costs. The Fortune 100 enterprise examined by the researchers had an original investment of $50 million in service parts inventories. Over a two-year period, the organization was able, with help of the analytical model, to dispose $13 million worth of service parts. This resulted in a tax savings of approximately $6 million alone.

3.2 "Hybrid" Models

Now our attention shall turn to a consideration of those analytical approaches combining the acquisition and disposal decisions. We note that the disposal decision now consists of the quantity of excess items to dispose, as well as the timing of disposals.

3.2.1 Acquisition and Disposal Models

Fukuda [11] was perhaps the first to jointly consider acquisition and disposal decisions. He examined ordering and disposal policies in a multiechelon, multiperiod inventory environment. Considering such details as ordering costs, disposal values, shortage penalties and holding costs, he was able to determine optimal policies for the planning horizon. Essentially, the decision made at the beginning of each time period was always one of the following: an order of a certain amount is placed, a given quantity is disposed, or no ordering or disposal choice is made (the "do nothing" alternative).

Rothkopf and Fromovitz [31] discussed the rental of container units, and the decision as to when to return the container to the supplier. They considered a commodity, purchased in bulk using containers that must be rented. Rental fees for the container stopped when the container is returned. However, returning the container (to terminate the rental charges) requires discarding unused contents. Under what circumstances, therefore, ought the container to be returned? The authors analyzed constant and exponentially distributed demand sizes, as well as the discounting of future costs. They further considered the decision as to the size of the container to rent.

Teisberg [36] illustrated a model to guide the ongoing acquisitions and disposals (releases) of the U.S. strategic petroleum reserve. He developed a multi-period, stochastic dynamic programming tool to analyze this situation, incorporating potential "states" of the oil market in a given time period. His methodology is from a rather "economics" viewpoint as he considered "consumer surplus" and the supply and demand functions for oil in both a domestic and world context. For each entering stockpile size and each possible oil market state, and using the present value of all relevant costs, he was able to determine the optimal stockpile acquisition or release rates for a specific time period.

The remainder of the articles in this section are in the context of "equipment replacement" models. As equipment ages and deteriorates, it may be replaced by newer and better machinery. Consequently, the older equipment could be considered "excess" to the current requirements of the organization. Annual maintenance and operating costs tend to increase as equipment ages. Salvage values tend to work in the opposite direction, as older equipment becomes less valuable. Note that, in this case, salvage values are not constant from year to year. Rather, they depend upon the year of asset disposition. However, during a given time period (ie. a year), these researchers have assumed that the salvage revenue is constant.

Waddell [40] analyzed the replacement of expensive highway tractors for Phillips Petroleum Company. At each node in his dynamic programming algorithm, the organization is faced with the following options: replace the equipment immediately, or retain it until the next opportunity for replacement. The basic question to be answered is whether or not a particular tractor ought to be replaced. If not now, then when should it be disposed? He used discounted cash flows to determine the effects of the respective policies, considering such factors as maintenance and operating costs, lease costs, license fees and road use taxes. He permitted savings due to investment tax credits and salvage values from surplus unit disposal. Use of his algorithm resulted in an annual savings of roughly $90,000.

Olorunniwo [26] proposed maintenance schedules, when the effects of such maintenance work were imperfect (in other words, such endeavours did not restore the equipment to a "good-as-new" condition). He determined the number and timing of preventive maintenance cycles that ought to be done before the equipment is overhauled. In addition, he suggested the number of overhaul cycles that should be done before the equipment is disposed (and new machinery is acquired). The author used the relative costs of various maintenance actions, the time value of money, and a Weibull failure probability distribution to generate the minimum cost maintenance, acquisition and disposal schedule.

A finite horizon, discrete-time, equipment (capacity) expansion and disposal model was developed by Rajagopalan and Soteriou [28]. Due to breakdowns caused by physical deterioration and technological obsolescence, the effective capacity of a piece of equipment may diminish over time. The authors assumed that capacity deterioration persisted at the same uniform rate each period for all equipment. The costs of capacity acquisition and usage consisted of initial purchase costs as well as operating and maintenance charges. Salvage value and operating costs depended upon the period of purchase and the number of periods of utilization. Capacity shortages were not permitted.

Since the decision variables (number of units of equipment to procure) are general integer types, they developed the following integer programming (IP) formulation (note that the structure is similar to that of a multi-dimensional knapsack problem):

$$Minimize\ Total\ Costs = \sum_{t=1}^{T} \sum_{j=t+1}^{T+1} \sum_{i=1}^{N} s_{itj} \quad X_{itj}$$

$$s.t. \sum_{t=1}^{\tau} \sum_{j=\tau+1}^{T+1} \sum_{t=1}^{N} a_{it\tau} X_{it\mathrm{T}} \geq d_{\tau} \quad \tau = 1,....,T$$

$$\text{and } X_{itj} \geq 0 \text{ and } X_{itj} \text{ integer} \quad \forall\, i,t \text{ and } j \geq t+1$$

where:

s_{itj} = net cost of purchasing a unit of equipment type i at the beginning of period t and disposing it at the beginning of period j ($> t$).
x_{itj} = number of units of type i acquired at the beginning of period t and disposed at the beginning of period j ($> t$).
$a_{it\tau}$ = effective capacity in period τ of equipment type i purchased in period t
d_{τ} = total demand for equipment capacity in period τ

The linear relaxation to the IP problem was used to obtain a lower bound for the value of the optimal solution. A heuristic interchange procedure, involving the rounding up of any fractional values from the preceding relaxation, was adopted as an efficient technique for obtaining a good feasible solution to the IP. In several computational experiments, this interchange strategy, used at the "root" node of a branch and bound tree, yielded excellent feasible IP solutions (% deviation of heuristic solution was about 0.6% from optimal value).

3.2.2 Quantity Discounts and Disposal Models

An additional class of models in which acquisition and disposal decisions are combined involves those approaches featuring quantity discounts and disposals. An important type of quantity discount treated in the literature concerns "all-unit" structures (see Johnson and Montgomery [17] or Silver, Pyke and Peterson [33] for a treatment of their effects on inventory management and control). An all-units discount, as opposed to an incremental one, offers the reduced cost on all procured units. Without loss of generality, one can assume a situation in which two unit prices are possible (c_0 at the lower quantity, and c_1 at the larger quantity, where $c_1 < c_0$). In order to take advantage of the discount, a certain number of units, Q_d, must be purchased.

Sethi [32] proposed certain situations in which it may be better to purchase the larger number of units (Q_d) at the lower unit price, then dispose of a given number (at, possibly, a cost to the organization). Obviously, the disposal of some units of the stock reduces inventory carrying charges.

Jucker and Rosenblatt [18] extended the work of Sethi. They evaluated the disposal of excess stock in a quantity discount context, for a single-period situation. They determined a range ($Q_r < Q_d$) just before the breakpoint, Q_d, such that it would be better for the purchaser to order the larger quantity. Certain claims regarding supplier behavior were illustrated. A "literal" supplier is one who charges c_0Q_r for orders of Q_r units, even though the purchaser could have acquired $Q_d > Q_r$ units for $c_1Q_d < c_0Q_r$. On the other hand, a "cooperative" supplier allows the purchaser to pay c_0Q_d, but only take delivery of Q_r units. Jucker and Rosenblatt also discuss the implication of probabilistic demand in this quantity discount and disposal decision. For stochastic usage, the purchaser will always wait until the end of the period to dispose excess units. This occurs due to the relative uncertainty surrounding total demand and since, in the single-period newsvendor formulation, the period is assumed to be so short in duration that holding costs may be ignored. In the deterministic case, the purchaser is indifferent as to the time of disposal.

In an effort to determine Q^* when faced with all-units price discounts, Gupta [14] developed upper bounds on the total annual relevant costs. He determined a relation that a specific price level must satisfy in order to yield the optimal procurement quantity. Should the price violate that relation, then it could be ignored from further consideration. This greatly reduced the computational effort required to find Q^*.

Arcelus and Rowcroft [1] examined the integration of purchasing and stock-control policies in the presence of secondary markets, a rather important practical problem that has received little attention in the academic literature. They considered both quantity and freight-rate discounts with the possibility of disposals. Their research, from somewhat of an economics angle, allows a price-dependent (downward-sloping) demand function. They assumed that there exists only one price break. Firms have the option of taking advantage of a larger quantity purchase, at lower unit costs. Since a constant "markup" is applied to purchase cost (to produce retail price), a lower unit cost will generate higher demand for the product. This is an extension over the work of Sethi. Arcelus and Rowcroft consider purchasing, ordering and holding costs. They derived the net profit resulting from the two alternatives: the "no discount" option, and the "discount" choice. Sensitivity analysis was performed to observe the effects of the various model parameters on resulting profit. A later article by the same authors [2] discussed multiple price breaks. For this problem scenario, a computationally efficient, simple-to-use two-stage algorithm was developed. First, one derived the solution for the generalized all-units discount structure, when disposals are not allowed. Then, the disposal decision was incorporated into the model. The profit or return on investment (ROI) of following each strategy was determined.

4. DIRECTIONS FOR FURTHER RESEARCH

This survey paper has discussed a number of methods for determining the optimal amount of excess stock to dispose. In the "hybrid" case, we have analyzed various procedures for jointly determining acquisitions and disposals. Although research in this problem area covers a span of almost half a century, there remain some key directions for further study. We shall illustrate four.

1. A greater effort ought to be placed at linking the acquisition and disposal decisions, rather than examining the disposal decision in isolation. The majority of articles in this paper that discussed this decision combination approached the topic from an "equipment replacement" context, or from a problem setting involving quantity discounts and concomitant disposals. From a practical standpoint, coupling the acquisition and disposal decision should be useful in situations wherein there exists highly uncertain demand for a product. Such a product could be one that is in the new product development or introduction phases of its life-cycle. A retailer

would be faced with the important decision as to how many units to order, and to dispose (should very low demand occur). Products prone to rapid technical obsolescence may also be appropriate candidates for these types of models. The organization would no longer have such a strong need for the product, but there may exist secondary markets in which the item may have some viability. Products subject to frequent shifts in consumer tastes, in which demand drops quickly, may also be suitable candidates for combined acquisition and disposal approaches.

From a project management perspective, the coupling of acquisition and disposal decisions could offer pertinent research opportunities. An appropriate decision scenario would be the construction of a large-scale project involving uncertainties regarding total requirements of an item. Given that the item has operational usage during the ongoing phase of the facility (obviously lower than total construction requirements), key decisions regarding procurement and disposal quantities would most likely need to be made.

2. Techniques for excess stock disposal have focussed on constant marginal salvage values. It may be worthwhile to determine the effect on acquisition and disposal policies should one consider non-constant marginal salvage values. At first glance, marginally diminishing disposal values may be appropriate. The more units an organization disposes, the less likely it is to receive the same unit salvage value for each disposal. However, one may be able to make a case for marginally <u>increasing</u> salvage values (Das [7] discusses solution of the EOQ under "price premiums", not price breaks). Market conditions and purchaser requirements enter the picture. This situation could result when a potential purchaser of excess stock is willing to pay a certain price for surplus units, provided a minimum number are transferred. Since, in this case, the purchasing organization is looking for a certain amount of stock, it would probably be unwilling to go to several different companies in order to procure its required quantity.

Consequently, up to a point, some purchasers may be willing to pay a greater marginal price for larger amounts of units available to be disposed.

3. Of the articles we presented, only a handful (e.g. Simpson, Tersine & Toelle, Rosenfield, Olorunniwo) offered some type of sensitivity analysis in their models. The ability to determine the relative effect of various values of model parameters on the overall solution is critical from an implementation perspective. Sensitivity analysis, definitely showcasing the important aspects of non-constant marginal salvage values, ought to be treated more extensively.

4. Most of the research discussed in this survey paper highlighted deterministic usage. More treatment of stochastic demand seems appropriate. In addition, using deterministic, time-varying demand would seem to be a good option for future models (only Hart, and Miller, Mellichamp and Henry discussed this important demand feature). The project management context discussed earlier would most likely involve time-varying usage. During the initial portion of the ongoing phase, higher-than-average usage would most likely occur (the so-called "infant mortality" syndrome). This would be followed by diminished usage, until such time as higher usage would commence due to aging and deterioration.

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