Disposal of excess stock at the end of a project when facing on-going operational usage

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Abstract

In this paper we consider an important decision faced in many large-scale construction projects. Specifically, there is a surplus stock of an item at the end of the construction phase. Moreover, the item is to be used (as a spare part or for routine replacement) in the on-going operations of the facility. There is an opportunity to dispose of some of the stock but the marginal salvage value is not necessarily constant. The decision variable is the disposal quantity. © 1999 Elsevier Science B.V. All rights reserved.

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1. Introduction

This paper examines an important materials management decision encountered in project management contexts. We explore the disposal of surplus stock after completion of the construction phase of a large-scale project. Specifically, there is excess inventory of an important, expensive item. This item will be used, as a spare part or for routine replacement, in the on-going operations of the constructed facility. There exists an opportunity to dispose of some of the stock; however, the marginal salvage value obtained for surplus disposals may not necessarily be constant. For example, one may earn a certain salvage value for, say, the first 10 units disposed, then a lower per unit salvage value for any disposals beyond that amount. (This is an example of diminishing returns where there is limited market interest in acquiring the units to be disposed.) This paper describes decision rules that one can implement to determine how much of the excess stock one should dispose, and the quantities one ought to retain.

Silver [1,2] conducted a survey of procurement and logistics managers involved in large-scale projects in the Province of Alberta (Canada). The disposal of surplus material after completion of a project was identified as one of the difficult decision areas faced by these managers; hence the need for a systematic method for tackling this problem.
Excess stocks may arise due to subsequent design changes in a facility, or to the inherent uncertainty in certain types of projects (e.g., subsurface work). Options for disposal include return to supplier (using a buyback clause in the purchase contract), sell (usually at a discount), or trade-in on a future purchase.

Several researchers have explored the disposal of excess stock. Each has attempted to determine an economic retention quantity (or time period) of stock. Disposal ought to occur only if inventory is in excess of this economic retention amount. The marginal salvage value for disposals has been assumed to be constant in all cases. Simpson [3] was an early contributor in this area. He developed a break-even relationship between the cost of retaining stock (storage and obsolescence costs) and the cost of disposing stock (repurchasing items in the future, if and when required). Mohan and Garg [4] and Kulshrestha [5] expanded Simpson’s results by allowing the probability of obsolescence to follow an exponential distribution, as opposed to the constant probability method used by the earlier author. Fukuda [6] examined disposal issues in a multiechelon inventory system. Hart [7] used a sequential search procedure to determine the economic retention quantity.

Tersine and Toelle [8] described excess inventory as a “dead weight” which uses valuable storage space, depletes working capital and diminishes one’s return on investment (ROI). The net benefit of excess item disposal was calculated as salvage revenue plus holding cost savings, less any future reorder and repurchase costs. The authors developed decision rules for the simple (no discounting of future costs) and present value (PV) (discounting) cases. As an outgrowth of their model, they computed the minimum economic salvage, the lowest unit price for which an item would be disposed.

Rosenfield [9] described the disposal of slow-moving inventory. He developed disposal rules for the case of stochastic demand and perishability. Finally, Stulman [10] carried out separate analyses for deterministic and Poisson demand. His net benefit equation included immediate salvage revenues, minus the PV of holding costs incurred in the usage of retained inventory, minus the PV of all future costs.

Section 2 describes the analytical model we have used to determine the net cost of disposing of excess stock. Section 3 then illustrates a decision rule for the disposal of excess stock when facing a constant marginal salvage value. Non-constant marginal salvage values are treated in Section 4. A summary is included in the final section.

2. The analytical model

Our approach is similar to that adopted by Stulman [10] in that we consider three distinct costs associated with the disposal/retention of excess stock. Specifically, we must incorporate:

1. salvage revenues (negative costs) associated with disposal,
2. inventory carrying costs incurred while retained stock is used (what we call the “transient” phase),
3. costs related to future replenishments in the on-going phase.

One determines the PV of all such revenues and costs in an attempt to determine the net cost of disposing a certain amount of excess stock. Disposing a larger amount of stock causes total salvage revenues to increase. Since disposals reduce retained stock, inventory carrying costs in the transient phase decrease. However, disposals cause the future replenishments in the on-going phase to occur sooner; hence, the PV of such costs increases. The trade-off becomes readily apparent since (1) and (2) are positively associated with disposals, while (3) moves the opposite direction.

The first step in determining the net cost of excess stock disposals is to develop a method for calculating the cost of future replenishments in the on-going phase. Since we are dealing with a very expensive item (e.g. compressor unit) with relatively low usage, we treat usage and the replenishment quantity as discrete variables. Thus, the pattern of replenishments in the on-going phase resembles a “step-wise” pattern.

We introduce the following notation:

- \( Q \) replenishment quantity,
- \( D \) demand rate,
- \( v_0 \) unit acquisition cost in the on-going phase,
Continuous discount rate (ie. a cost of \( x \) incurred at time \( t \) has \( PV = x \exp(-xt) \)),

Out-of-pocket costs (other than cost of capital) per unit time of carrying one dollar of inventory,

Fixed cost associated with each replenishment,

\( Z(Q) \) PV of all future costs in the on-going phase.

The following assumptions are also made:

1. The demand rate, \( D \), is known and constant,
2. There is no item obsolescence.

The stepwise pattern in the on-going phase implies that one unit is carried from time 0 (the start of a cycle) to time \( 1/D \), one unit is carried from time 0 to time \( 2/D \), and so forth. Out-of-pocket carrying costs are continuously discounted. In a fashion analogous to that first used by Hadley [11], we obtain that the present value of the infinite stream of stepwise patterns is

\[
Z(Q) = \frac{A + Qv_o(1 + (i/z))}{1 - \exp(-Qx/D)} \frac{iv_o\exp(-x/D)}{x[1 - \exp(-x/D)]}
\]

Gurnani [12] also illustrates the use of continuous discounting.

In order to adopt the optimal replenishment strategy in the on-going phase, we must select the integer \( Q \) that minimizes \( Z(Q) \). We have been able to prove [13] that the second difference of \( Z(Q) \) is positive, hence that \( Z(Q) \) is a convex function in \( Q \). As a result, a best value of \( Q \), \( Q^* \), is the smallest \( Q \) such that \( Z(Q + 1) - Z(Q) > 0 \), i.e.

\[
\exp\left(\frac{Qz}{D}\right) > 1 + \left[\frac{1 - (z/2D)A}{1 + (i/z)}\right]
\]

\[
+ \left(1 - \frac{z}{2D}\right)\left(\frac{Qz}{D}\right).
\]

We observe that the left-hand side of Eq. (2) is exponentially increasing in \( Q \) (from a starting point of 1), while the right-hand side is linear in \( Q \) (it starts above 1). Thus, there will be one and only one place at which the two functions intersect. The smallest integer \( Q \) greater than this intersection point is \( Q^* \). There is, then, a relatively straightforward search involved in finding \( Q^* \). As an aside, we note that the EOQ (using \( z + i \) as a carrying charge) is a very good point at which to begin the search.

As a numerical example, assume the following values:

\( D \) 20 units/y,
\( A \) $100,
\( z \) 0.10/y,
\( i \) 0.05 $/S/y,
\( v_o \) $200/unit.

We find that \( Q^* = 11 \) units, and that \( Z(Q^*) = $43585.47 \).

Now, we shall address the costs associated with disposing a certain amount of the excess stock. We shall require the following notation:

\( I \) initial inventory at the end of the construction phase (but before any disposal decision),
\( W \) disposal quantity,
\( M \) inventory remaining after any disposal decision (equivalent to \( I - W \)),
\( v_c \) unit value of remaining inventory (it was acquired in the construction phase; hence, the subscript c),
\( g \) marginal salvage value (\( g \) could be negative, if there exists a net cost in disposing a surplus unit).

We wish to choose the value of \( W \) (or, equivalently, \( M \)) which minimizes the PV of total discounted costs. As described earlier, these costs may be represented by three separate items. We have

1. Revenue (negative cost) from surplus disposal. This is given as \( -gW \).
2. Inventory carrying charges during the transient phase. These will be calculated in a manner similar to that used in evaluating the carrying charges in the on-going phase.
3. Discounted present value of the costs of all replenishments in the on-going phase. If \( M \) units are retained, the future replenishments are delayed by \( M/D \) time units.
The net cost equation thus becomes

\[ PV(M) = -gW + iv_e \left[ \frac{M}{x} - \exp\left( -\frac{x}{D}\left( 1 - \exp\left( -\frac{Mx}{D}\right) \right) \right) \right] + \exp\left( -\frac{Mx}{D}\right)Z(Q^*). \]

Since \( W = I - M \), we may rewrite Eq. (3) as

\[ PV(M) = -gI + gM + iv_e \left[ \frac{M}{x} - \exp\left( -\frac{x}{D}\left( 1 - \exp\left( -\frac{Mx}{D}\right) \right) \right) \right] + \exp\left( -\frac{Mx}{D}\right)Z(Q^*). \]

Again we have proved \([13]\) that \( PV(M) \) is convex in \( M \) (an alternate approach would be to use quasiconvex functions as outlined in Avriel et al. \([14]\)). Thus, a best \( M \), denoted by \( M^* \), is the smallest integer \( M \) such that \( PV(M + 1) - PV(M) \geq 0 \). We obtain that \( M^* \) is the smallest integer \( M \) which satisfies

\[ M > \frac{D}{x} \ln \left[ \frac{iv_e \exp\left( -\frac{x}{D}\right) + Z(Q^*)\left( 1 - \exp\left( -\frac{x}{D}\right) \right)}{g + iv_e/x} \right]. \]

We observe that, as \( g \) decreases, \( M^* \) will tend to increase. This is logical, for the less we earn for the disposal of salvage units, the more we are likely to retain in inventory.

3. Decision rule – constant marginal salvage value

Now that we have obtained expressions for the relevant net costs, we can develop a decision rule for the disposal of excess inventory. The \( M^* \) value calculated in Eq. (5) above is equivalent to the “economic retention quantities” described in the literature. For the case of a constant marginal salvage value, the optimal decision is to retain any surplus inventory up to and including the \( M^* \) amount. We dispose of any stock in excess of \( M^* \). Let us expand on the numerical example presented earlier. In particular, let:

- \( v_c \) $100/unit (note that, due to quantity discounts, unit acquisition costs in the construction phase will most certainly fall below acquisition costs in the on-going phase),
- \( g \) $30 per unit.

Then, \( M^* = 242 \) units. As a result, the decision rule becomes:

For \( I \leq M^* = 242, W^* = 0 \) (i.e. dispose nothing).
For \( I > M^* = 242, W^* = I - 242 \).

4. Decision rule – non-constant marginal salvage values

Previous excess stock disposal research has assumed a constant marginal salvage value. In practice, however, it may not be possible to earn as much for the 100th unit disposed as for, say, the 10th. One is likely to observe diminishing marginal salvage values as the disposal quantity is increased. A commonly occurring situation is likely to be one in which a certain salvage value (\( g_1 \)) is earned for a specific number of units disposed, then a lower salvage value (\( g_2 \)) for units disposed beyond that amount (up to a certain limit), then an even lower salvage value (\( g_3 \)) for disposals beyond the maximum possible with \( g_2 \), and so on. We thus define:

- \( g_i \) marginal salvage value earned for \( i \)th level of disposals,
- \( N_i \) maximum number of disposals possible for \( g_i \),
- \( M_i^* \) the best remaining stock level (after possible disposal) when there is a marginal salvage value \( g_i \).

The variable \( M_i^* \) is calculated by replacing \( g \) with \( g_i \) in Eq. (5). We thus have that \( M_i^* \) is the smallest integer \( M \) such that

\[ M > \frac{D}{x} \ln \left[ \frac{iv_c \exp\left( -\frac{x}{D}\right) + Z(Q^*)\left( 1 - \exp\left( -\frac{x}{D}\right) \right)}{g_i + iv_c/x} \right]. \]

Since \( g_1 > g_2 > \cdots > g_m \) it follows that \( M_1^* \leq M_2^* \leq \cdots \leq M_m^* \). Strict inequalities do not necessarily apply because of the integer requirement on \( M_i^* \).
The first aspect of this problem is simple to analyze. For any $I$ (inventory left prior to disposal) \( \leq M^*_1 \), $W^* = 0$ (i.e. there is no disposal). We know that the most we can dispose at a revenue rate $g_1$ per unit is $N_1$ units. As a result, for $M^*_1 < I \leq M^*_1 + N_1$, $W^* = I - M^*_1$ (we dispose the excess above $M^*_1$).

Disposals at $g_2$ per unit will only become attractive if the remaining inventory exceeds $M^*_2$. Thus, until $I - N_1 > M^*_2$, we will only dispose the $N_1$ units (at $g_1$ per unit). We will dispose excess stock at $g_2$ per unit when $M^*_2 + N_1 < I \leq M^*_2 + N_1 + N_2$. Fig. 1 shows a pictorial representation of the optimal disposal policies, when facing diminishing marginal salvage values. We obtain an alternating pattern of plateaus (no additional disposals are taking place) and ramps (additional disposals are occurring at a certain $g_j$). The plateau between the use of $g_i$ and $g_{i+1}$ is of width $M^*_{i+1} - M^*_i$, and the ramp for $g_i$ continues for $N_i$ units. We note that there will be no plateau between two ramps if and only if $M^*_{i+1} = M^*_i$.

Defining $M^*_0 = 0$ and $N_0 = 0$, we can develop the following decision rules:

**PLATEAU:**
For $M^*_i + \sum_{j < i} N_j < I \leq M^*_i + \sum_{j < i} N_j$, \[ W^* = \sum_{j < i} N_j. \]

**RAMP:**
For $M^*_i + \sum_{j < i} N_j < I \leq M^*_i + \sum_{j < i} N_j + N_i$, \[ W^* = I - M^*_i. \]

**5. Summary**

When faced with excess stock at the completion of the construction phase of large-scale projects,
materials managers must make important decisions regarding retention and disposal of surplus items. Despite the fact that non-constant marginal salvage values add complexity to the problem, an efficient procedure has been developed to determine optimal disposal policies in this decision-making environment.

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References


