

Preferred Scenarios in the Sport of Curling

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In the sport of curling, participants continually debate this question: In the latter stages of a game, is it better for a team to be trailing by one point yet have the last shot, or ahead by one point without the last shot? Based on results from over 900 Canadian men's national championship games, we found that it is better for a team to be ahead by one point without the last shot. We surveyed world-class curlers to determine their preferences and probability estimates for given outcomes and discovered that several of these participants have disparate preferences and probability estimates.

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In a variety of sports, people hotly contest the attractiveness of different scenarios, strategies, or decisions. For example, should a team intentionally walk a batter in the latter stages of a baseball game (Steinsaltz 1999)? Brimberg and Hurley (1999) used game theory to determine whether a soccer player should ever aim the ball directly at the keeper in a soccer penalty kick. Both Porter (1967) and Sackowitz (2000) examined point-after-touchdown decisions in football. They suggested scenarios under which it would be appropriate to try for the (more difficult) two-point conversion rather than the "safe" one-point attempt. In ice hockey, a team may decide to replace its goalie with an attacking skater, particularly if the team is trailing by a single goal late in the game (Morrison and Wheat 1986).

We developed a quantitative model of a classic question in the sport of curling and surveyed several world-class curlers to determine their preferences regarding particular scenarios.

The Sport of Curling

Curling is a winter team sport with origins in 17th-century Scotland. The first curling club in North America was founded in Montreal in 1807. Curling's roots in the United States began with the founding of the Orchard Lake Curling Club (near Detroit) in 1832. Currently, over 15,000 curlers participate in 135 active curling clubs in 26 US states. The primary pockets of participants are in Wisconsin, North Dakota, and Minnesota. Curling continues to be a very popular sport in Canada and several European countries (for example, Switzerland, Scotland, Norway, and

Sweden). Participation is growing in the Pacific Rim region, and Japan has begun to field world-class teams. Today, curling is played by millions of people in 33 countries. Murray (1981) and Weeks (1995) provide thorough historical discussions of curling.

Essentially, curling resembles shuffleboard on ice. The game is played indoors on sheets of ice 14 feet wide and 146 feet long. At each end of the sheet is a *house*. The house is a set of four concentric circles of various diameters (12 feet, eight feet, four feet, and one foot).

Curling is played with circular disks of polished granite ("rocks") weighing approximately 45 pounds each. The teams (four players to a side) take alternate turns sliding these rocks down the sheet of ice. To slide the rock, a player places a foot in a rubber foothold known as a *hack*. Then, the player pushes off from the hack and, while gliding along the ice, releases the rock towards the target house at the far end of the curling sheet. Two other members of the team shuffle alongside the rock, brushing the ice in front of the curling stone. Brushing has a two-pronged effect; namely, helping the rock to travel both further and straighter by creating a thin film of water under the rock (in fact, good sweepers can increase the distance a stone travels by as much as 15 feet). The remaining member of the team stands in the target house at the far end of the curling sheet. He or she indicates the target for the person sliding the rock and instructs the two brushers (often quite vigorously!) as to the appropriate amount of brushing. Generally, teams attempt to strategically position their rocks within the house. However, a team may try to place a curling stone in front of the house (thereby

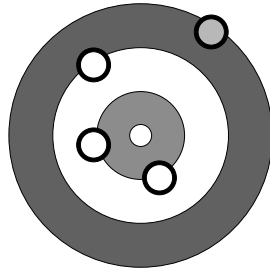


Figure 1: The curling team playing the lighter-shaded rocks scored three points because all of its rocks were closer to the center of the house than the opposition’s (darker-shaded) rock.

guarding one of its rocks in the house) or attempt to remove the opposition’s rock(s) from play. This latter shot is termed a take-out.

An end is completed when each team has played its eight rocks (the four players on a team each slide two rocks). The term *end* is derived from the fact that scoring takes place at alternating ends of the ice sheet. Upon the completion of an end, the score is tallied. A team scores a point for each stone closer to the middle of the house than the opposition’s closest stone. For example, suppose Team A has one rock in the house and Team B has three. If all of Team B’s stones are closer to the middle of the house than Team A’s one stone, Team B scores three points (Figure 1). Conversely, if Team A’s rock is closer to the middle of the house than any of Team B’s rocks, then Team A scores one point (Figure 2). A blank end occurs when no rocks are in the house at the completion of an end. A standard curling game consists of 10 ends. If the game is tied after 10 ends, the teams play extra ends until one of them scores at least one point. The scoreboard used during a curling game shows the scores for all 10 ends and the total (final) score (Figure 3).

As one might expect, the team that has the last rock in an end has a strategic advantage (similar to having

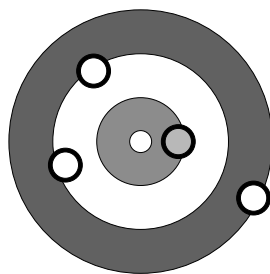


Figure 2: The curling team playing the darker-shaded rocks scored one point because its rock was closer to the center of the house than any of the opposition’s (lighter-shaded) rocks.

	1	2	3	4	5	6	7	8	9	10	Total
Team X	0	0	2	0	0	2	0	1	1	0	6
Team Y	1	0	0	1	1	0	2	0	0	2	7

Figure 3: The scoreboard for a curling game shows how many points each team scored in each end. In this example, Team Y defeated Team X by a score of 7 to 6. Team Y scored one point in the first, fourth, and fifth ends and two points in the seventh and 10th ends. Team X scored two points in the third and sixth ends and one point in the eighth and ninth ends. The second end is a blank end because neither team had rocks in the house upon completion of the end.

the final at-bat in baseball or the last shot in basketball). In the jargon of curling, this is known as “having the hammer.” Teams flip a coin to determine who will have the last rock in the first end. As the game progresses, the team with the lowest score in the previous end gets the hammer in the next end. Should a blank end occur, the team that had the last shot retains the hammer for the subsequent end.

A classic question that curlers often debate is the following: Entering the 10th end (the final end of a regulation game), is it better for Team A to be up by one point without the hammer, or down by one point with the hammer? In the former scenario, Team A is leading in the game but its position may be somewhat precarious (because it does not have the final shot in the end). Although the latter scenario has Team A trailing its opposition, it may derive some confidence from being able to play the final rock.

Analytical Model

Our approach was to determine the probability of victory for each of the two scenarios. These probabilities may be considered expected values, because a win is “worth” one point while a loss has no value. We did not develop any new methodology; we simply applied a quantitative tool to a rather interesting problem. We use the following general notation to represent the probability of scoring a certain number of points in a particular end, with or without the hammer: $P\{X = k \mid e, h\}$, where

- X = a random variable representing the scoring in a curling game,
- k = the particular number of points scored,
- e = the specific end under consideration, and
- h = whether or not Team A has the hammer.

We use $\sim h$ to represent the event of Team A not having the hammer. For example, $P\{X = 2 \mid 10, \sim h\}$ represents the probability of Team A scoring two points in the 10th end given that it does not have the hammer. The notation $P\{X = -1 \mid 10, h\}$ denotes the probability that Team A gives up a single point (that

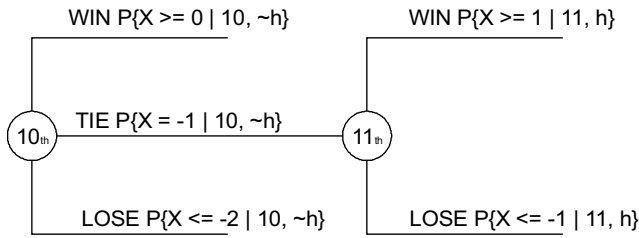


Figure 4: If Team A is ahead by a single point without the last shot in the 10th end, it can win the game by scoring at least one point in the 10th end or by the end being blanked (neither team scoring). We depict these possibilities in the top branch of the 10th end node. The game is tied if the opposition scores a single point (middle branch). Team A loses if the opposition scores at least two points in the 10th end (bottom branch). Should the game be tied, the teams play an extra end and Team A has the hammer (the last turn) in the extra end. The team that scores in the extra end wins the game.

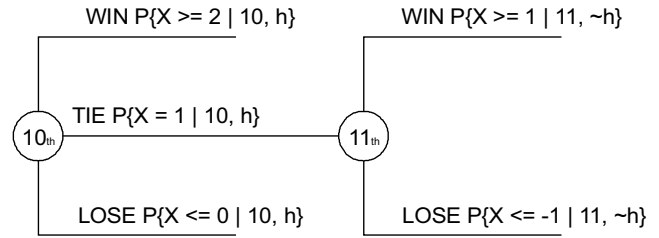


Figure 5: When Team A has the hammer (the last turn) and trails by a single point, it wins the game by scoring at least two points in the 10th end (top branch). The game is tied if it scores a single point (middle branch). It loses if the end is blanked, or if the opposition scores (bottom branch). As we showed in Figure 4, the teams play an extra end if the game is tied after the 10th end. Because Team A would have scored in the 10th end (to create the tie), it would not have the hammer in the extra end.

is, Team B scores one point) in the 10th end given that Team A has the hammer.

We begin by exploring the scenario of Team A entering the 10th end ahead by a single point without the hammer (Figure 4). We need to consider all cases that would give Team A a victory. At the very least, Team A could win by “blanking” the 10th end of play (no points are scored). It would also emerge victorious by scoring at least one point in this end. This situation can be succinctly represented by $P\{X \geq 0 \mid 10, \sim h\}$. Team A could also give up a single point in the 10th end (to tie the game), whereupon the teams would play an extra (11th) end. If Team A scored at least one point in this 11th end, it would win the game. Because Team B scored in the 10th end, Team A would have the hammer in the 11th end ($P\{X = -1 \mid 10, \sim h\} \times P\{X \geq 1 \mid 11, h\}$). The 11th end could be blanked (for example, if the team with the hammer did not place its final rock in the house). If this occurred, the teams would play a 12th end. However, a blanked 11th end is extremely rare. Thus, the expected value of winning when entering the 10th end ahead by a single point without the hammer is $E(UP) = P\{X \geq 0 \mid 10, \sim h\} + P\{X = -1 \mid 10, \sim h\} \times P\{X \geq 1 \mid 11, h\}$.

In the case in which Team A entered the 10th end trailing by a single point with the hammer (Figure 5), Team A could win by scoring at least two points in the 10th end ($P\{X \geq 2 \mid 10, h\}$). The game would be tied after 10 ends of play if Team A scored a single point in the 10th end. Team A would then not have the hammer in the 11th end but would win the game by scoring at least one point. Consequently, we would have $P\{X = 1 \mid 10, h\} \times P\{X \geq 1 \mid 11, \sim h\}$. The expected value of winning when entering the 10th end trailing by a single point with the hammer is $E(DN) = P\{X \geq 2 \mid 10, h\} + P\{X = 1 \mid 10, h\} \times P\{X \geq 1 \mid 11, \sim h\}$.

If the expected value of winning when entering the 10th end ahead by a single point without the hammer

exceeds the expected value of winning when entering the 10th end trailing by a single point with the hammer, then the preferred scenario for Team A would be to enter the 10th end ahead by one without the hammer. It would prefer the other scenario, to enter the 10th end trailing by one point with the hammer, if this expected value exceeded the expected value of winning when entering the 10th end ahead by one point without the hammer.

To empirically determine the preferred scenario, we gathered statistical information recorded from 1985 to 1997 at the Canadian Men’s Curling Championships (also known as the Brier) (Table 1).

We can obtain probabilities of scoring a certain number of points in a particular end, with or without the hammer, from the frequency table. For example, this table suggests that a team has an 80/410 (19.51 percent) chance of scoring exactly two points in the 10th end given that it has the hammer. To obtain the probability of scoring without the hammer, we simply note that the probability of scoring a certain number of points in a specific end with the hammer equals the probability of giving up a certain number of points in a particular end without the hammer. Moreover, the probability of scoring a certain number of points in a given end without the hammer equals the probability of giving up a certain number of points in a particular end with the hammer. For instance, there is a 2/118 (1.69 percent) chance that a team with the hammer will score exactly three points in the 11th end. This particular probability corresponds to the exact likelihood that a team will give up three points in the 11th end without the hammer.

A team may concede defeat at any point in the game if it believes its opponent’s lead is insurmountable. Of the 902 games that completed at least four ends, less than half (410) went the full 10 ends.

	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	
1		1	4	25	104	251	322	156	32	7			902
2		2	6	53	133	184	322	162	32	6	2		902
3		2	12	56	161	163	277	170	51	8	1	1	902
4		4	11	54	149	154	294	180	49	6	1		902
5	1	1	14	42	156	135	297	189	50	10	4	1	900
6	1	5	9	46	154	149	292	170	54	12	4		896
7	1	3	6	55	133	154	282	179	46	14	1		874
8	1	4	8	44	147	143	280	168	42	6	3		846
9			14	35	124	164	231	152	39	4	1	1	765
10		1	6	9	75	22	200	80	12	4	1		410
11				4	15	4	85	7	2	1			118
12							3	1					4
Total	4	23	90	423	1,351	1,523	2,885	1,614	409	78	18	3	8,421

Table 1: The rows of this frequency table represent the ends; the bottom row is the sum of each column. Each column represents the number of points scored in an end with the hammer (the last turn). The frequency table shows that scores of +1, +2, and 0 points are most common. “Stealing” (scoring by a team without the hammer) occurs, but the likelihood of a team stealing two or more points is small. Further, a team may concede the game if it believes its opposition’s lead is insurmountable. Of the 902 games for which we collected our data, only 410 went the full (regulation) 10 ends. An 11th end was played in 118 games, and a 12th end was required on only four occasions.

The 11th end may be blanked, in which case the teams would play another end. The team that had the hammer in the 11th end would retain it for the 12th end. However, of 902 games played in the Brier, teams played a 12th end only four times. We can simply lump those four 12th end games in with the 11th end data. Although we could have added the possibility of an 11th end blank to our model, its occurrence was so rare that we thought it did not merit consideration. On the whole, it would have added little to our overall analysis.

The 410 games that went to the 10th end include all games that went at least 10 ends, no matter the score differential as the 10th end began. Our objective was to analyze those scenarios in which teams either trailed or led by a single point in the 10th end. Therefore, we reduced our data to only consider those games in which the two teams’ scores differed by exactly one point after nine ends of play (Table 2).

Using Table 2, we obtained empirical values for the expected value of winning when entering the

10th end ahead by a single point without the hammer and the expected value of winning when entering the 10th end trailing by a single point with the hammer ($E(UP)$ and $E(DN)$):

$$E(UP) = 61/221 + 113/221 \times 65/76 = 0.713, \quad \text{and}$$

$$E(DN) = 47/221 + 113/221 \times 11/76 = 0.287.$$

Therefore, the preferred scenario based on 13 years worth of data from the Canadian Men’s Curling Championship is to be ahead by one point without the hammer.

The events of winning the game in the extra end, with or without the hammer, are complementary. We use $P\{X \geq 1 \mid 11, h\}$ and $P\{X \geq 1 \mid 11, \sim h\}$ to represent these events, respectively. Given the 10th end tallies recorded in Table 2, we assessed the particular value of $P\{X \geq 1 \mid 11, h\}$ that would make one indifferent between the UP or DN scenarios. Our indifference probability is 0.438. Therefore (given our observed 10th end point totals), curlers ought to prefer the UP scenario as long as they feel their probability of winning the game in the extra end with the hammer is at least 43.8 percent.

Survey Methodology

To obtain expert opinion regarding the preferences between these two scenarios, we sent a short e-mail survey (Appendix) to members of the World Curling Tour (WCT). As a member of the WCT, one of the authors received a list of their e-mail addresses. The WCT was founded in 1990 with the simple philosophy of organizing a championship for elite curlers.

	-4	-3	-2	-1	0	1	2	3	4	5	
10	1	5	4	39	12	113	34	8	4	1	221
11			2	9	4	55	4	1	1		76
12						3	1				4

Table 2: This frequency table shows the tallies in the 10th, 11th, and 12th ends for those games with a score differential of strictly one point after nine ends of play. We used these values to determine the expected value of winning when entering the 10th end ahead by a single point without the hammer ($E(UP)$) and the expected value of winning when entering the 10th end trailing by a single point with the hammer ($E(DN)$).

	Respondent #1	Respondent #2
#1 (preferred scenario)	UP	DOWN
#2	9	8
#3	3	3
#4(i)	5	4
#4(ii)	4	5
#4(iii)	1	1
#5(i)	4	2
#5(ii)	5	6
#5(iii)	1	2
$E(UP)$ calculation	0.850	0.680
$E(DN)$ calculation	0.620	0.550
Agreement?	YES	NO

Table 3: Two respondents, both former world curling champions, provided their preferences and probability estimates. Respondent 1’s preferred scenario, being one up without the last shot, is supported by his probability estimates because the expected value of winning when entering the 10th end ahead by a single point without the last shot ($E(UP)$) exceeds the expected value of winning when entering the 10th end trailing by one point with the last shot ($E(DN)$). However, respondent 2’s preference conflicts with his expected value calculations.

Its mission is “to promote the sport of curling on a global basis by providing a first-class Tour and Championship” (www.wctour.com).

In our survey, we first asked respondents for their preference for either scenario. The answers to questions 2 through 5 allowed us to compute specific values for $E(UP)$ and $E(DN)$ for each curler. We could then compare these values to the curlers’ indicated preference, thus determining whether curlers’ expected values were congruent with their desired scenarios.

We computed values for $E(UP)$ by multiplying the answers to question 5ii and question 2, and then adding this product to the answer to question 5i (we divided the answers by 10 to create expected values between 0 and 1). Similarly, we calculated the values for $E(DN)$ by adding the answer to question 4i to the product of the answer to 4ii and the response to question 3. We did not use the responses to questions 4iii and 5iii in calculating expected values because they concern losing curling games. We included these questions to help survey respondents to divide their curling experiences into the three possible outcomes (win, tie, loss). We compared the responses received from two curlers (both former world champions) (Table 3). The first respondent’s preference was congruent with the probabilities, while the second’s was not.

Results

We were pleased with the number of responses to our survey. We sent it to over 250 members of the WCT and received 113 responses. A fair number of the surveys were bounced because the e-mail addresses were inaccurate.

We were tremendously encouraged by the number of respondents who expressed great interest in our questions and eagerness to learn of the eventual results. Moreover, we obtained responses from many former world curling champions. We show their countries of origin and year(s) of championship:

- Kerry Burtnyk (Canada, 1995),
- Rick Folk (Canada, 1980, 1994),
- Bert Gretzinger (Canada, 1994),
- Peter Lindholm (Sweden, 1997),
- Ed Lukowich (Canada, 1986),
- Hammy McMillan (Scotland, 1999),
- Ron Mills (Canada, 1980),
- Linda Moore (Canada, 1985),
- Brent Pierce (Canada, 2000),
- Jeff Stoughton (Canada, 1996),
- Magnus Swartling (Sweden, 1997).

With such a large contingent of former world champions among our respondents, we had the utmost confidence in our survey results.

Our purpose was to explore the respondents’ preferences regarding the particular scenarios and then to compare their expected values to their preferred scenarios. Sixty-six of our respondents (58.4 percent) said they preferred to be trailing by one point with the hammer in the 10th end. The remaining 47 (41.6 percent) wanted to be ahead by a single point without the hammer. Although the majority favor the down scenario, the issue is by no means one-sided. Our survey of elite curlers shows that considerable debate exists concerning the preferred scenario.

The members of the up contingent are far more consistent in their preferences than their down counterparts. Of the 47 respondents who indicated that “up without the hammer” was their preferred scenario, 42 of them (89.4 percent) provided probability estimates in which up was favored over down ($E(UP) > E(DN)$). For the 66 who preferred “down with the hammer,” only 31 of them (47.0 percent) had probability estimates favoring down over up ($E(DN) > E(UP)$). The other 53.0 percent of this group had probability estimates that disagreed with their indicated preference.

Although several respondents preferred to be one down with the hammer in the 10th end, their probability estimates for different scenarios showed that their preferred scenario was not in line with their probability estimates. Their hearts were ruling over their heads. They may be the aggressive gamblers in the sport, wanting the challenge of coming from behind late in the game. Those who preferred being ahead by one point without the hammer were far more consistent. Their hearts agreed with what their heads were telling them.

Appendix

Curling Survey

Dear {respondent's name}

A colleague and I are interested in statistically analyzing some of the key aspects in the sport of curling. In particular, we are attempting to determine whether teams would prefer to be 1 up without the hammer after nine ends of play, or 1 down with the hammer after the 9th end.

Recognizing your tremendous experience and understanding of curling, we would appreciate a few minutes of your time to respond to the following questions.

In each of these questions, you may consider that you are playing an opposing team of equal caliber to yours. Place your responses inside the box.

1. Generally, which situation would you prefer:
 - i. You are playing the 10th end and are leading by 1 without the hammer?
 - ii. You are playing the 10th end and are trailing by 1 with the hammer?
2. Out of 10 extra end games, how often would you expect to win in the extra end with the hammer?
3. Out of 10 extra end games, how often would you expect to win in the extra end without the hammer?
4. Suppose that, as the 10th end begins, you are trailing by 1 with the hammer. Out of 10 games, how often would you
 - i. Expect to win in the 10th end (i.e., you scored more than 1 point)?
 - ii. Expect to play an extra end (i.e., you only scored 1 point)?
 - iii. Expect to lose in the 10th end (i.e., you did not score at all)?

NOTE: The sum of these three boxes should be 10.

5. Suppose that, as the 10th end begins, you are leading by 1 without the hammer. Out of 10 games how often would you
 - i. Expect to win in the 10th end (i.e., you blanked or scored in 10)?
 - ii. Expect to play an extra end (i.e., your opposition scored 1 point)?
 - iii. Expect to lose in the 10th end (i.e., your opposition scored 2 or more)?

NOTE: The sum of these three boxes should be 10.

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