The Cost of Capital and Optimal Financing Policy in a Dynamic Setting

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Abstract

This paper revisits the Modigliani-Miller propositions on the optimal financing policy and cost of capital in a dynamic setting. In an environment without taxes and bankruptcy costs, the results are generally consistent with Modigliani and Miller Propositions 1 and 2. However, the first proposition should be presented and interpreted more carefully, as given firm characteristics, there is only one optimal capital structure. Thus, a firm’s capital structure is relevant. A relaxation of assumptions about either taxes or bankruptcy costs leads to conclusions that are generally different from those in Modigliani and Miller (1958).

Key words: Capital structure; Cost of capital; Time preferences.

JEL classifications: D21; D58; G32.
1 Introduction

What is the optimal capital structure of a firm? The seminal paper by Modigliani and Miller (1958) shed some light on this issue using a static partial equilibrium model and assuming that:

1. capital markets are perfect;
2. there are no arbitrage opportunities;
3. individual investors can borrow at the same interest rate as firms;
4. expected earnings before interest and taxes (EBIT) do not depend on the firm’s capital structure;
5. there are no taxes and no bankruptcy costs, that is, the cost of debt does not depend on the firm’s leverage.

Modigliani and Miller (1958, 1963) reach several fundamental conclusions on the company’s capital structure and cost of capital:

1. The “market value of any firm is independent of its capital structure and is given by capitalizing its expected return at the rate \( \rho_k \) appropriate to its class. \(<\ldots>\) the average cost of capital, to any firm is completely independent of its capital structure and is equal to the capitalization rate of a pure equity stream of its class.” This is known as Modigliani and Miller Proposition 1.\[1\]

2. Return on equity capital of the levered firm is equal to the sum of the weighted average cost of capital (WACC) of the unlevered firm and the product of the debt-to-equity ratio and the difference between the WACC of the unlevered firm and cost of debt (Modigliani and Miller Proposition 2).

3. If there are taxes, the optimal capital structure is 100% debt and the WACC decreases with leverage.

4. If there are bankruptcy costs, Modigliani and Miller Proposition 1 is unaffected;
however, the return on equity capital of the levered firm increases with the debt-to-equity ratio but at decreasing rate (rather than constant as in Modigliani and Miller Proposition 2).\footnote{Modigliani and Miller (1958) explain graphically this statement rather than prove analytically.}

The first three conclusions have been recognized as the foundation of modern corporate finance, and each student enrolled in an undergraduate finance degree or MBA program is required to learn them.

Modigliani and Miller (1958, 1963) as well as the later studies (for example, Kraus and Litzenberger (1973)) ignore the dynamic nature of a firm. The authors employ a static partial equilibrium model. There are at least two key differences between static and dynamic models. Firstly, static models do not include the firm’s manager’s time and risk preferences that impact her production, investing, and financing choices. We do not directly observe the time and risk preferences of the firm’s manager; however, they can be instrumented by executive compensation structure as it is used to align the managerial incentives with shareholders’ interests. Several empirical studies report the significant relation between CEO compensation structure and firm’s capital structure. Agrawal and Mandelker (1987) find that firms whose managers have more equity incentives, rely more on long-term debt and preferred stock financing following an acquisition or a sell-off. Mehran (1992) reports that firms that grant their CEOs more stock options have more long-term debt. Ortiz-Molina (2007) argues that pay for performance sensitivity increases with convertible debt but decreases with straight debt. Bhagat, Bolton, and Subramanian (2011) find that long-term debt-to-assets and short-term debt-to-assets ratios decrease with the manager’s cash compensation. Another stream of literature examines the impact of equity based compensation on firm risk. Guay (1999) reports the positive impact of the sensitivity of CEO wealth to stock volatility on stock return volatility. Rajgopal and Shevlin (2002) provide evidence that stock options encourage managers to invest in risky projects. Coles, Daniel, and Naveen (2006) find that managerial compensation structure with higher sensitivity of
CEO wealth to stock volatility and lower CEO pay-performance sensitivity lead to higher total debt-to-assets ratio. Thus, we can conclude that the manager’s incentives and preferences are important determinants of her long-term financing and investing decisions. Time preferences determine how earnings are distributed over time. A higher subjective discount rate (or less patient manager) means that more earnings will be generated in the earlier periods. Risk preferences show the magnitude of a manager’s response to a shock. The response of a more risk-averse manager to a shock will be weaker and slower. Models without time and risk preferences might imply suboptimal financing and investing decisions. Secondly, static models do not explicitly include a production function with physical capital and labor (and/or raw materials) as the inputs. Instead, they assume that a firm generates a certain stream of earnings. However, there are an infinite number of different combinations of capital stock and other production factors that generate the same level of earnings. Capital stock can be used to determine firm value; therefore, static models without explicit production function might be not able always properly assess firm value.

In this paper, I use a dynamic partial equilibrium model to analyze whether the conclusions of Modigliani and Miller (1958) hold in a dynamic environment. The model replicates the life and simplified behavior of a representative firm in a dynamic world and is based on the following assumptions:

- a firm’s manager maximizes a certain objective function that positively depends on shareholder value;
- in each time period, the firm’s manager makes several simultaneous decisions, specifically, how much capital to raise in the external equity and debt markets, how much to produce, and how much to invest in capital stock;
- a firm uses a mix of equity and debt to finance its activities;
- debt has advantages (such as tax deductibility of interest expenses and lower costs)
and disadvantages (increased bankruptcy risk).³

• there are no agency costs of debt;

• a firm produces a single tradable final good that is sold in a competitive market;

• the relation among all endogenous variables and their dynamics are jointly determined in equilibrium.

First, I consider an environment without taxes and bankruptcy costs. I show that a firm’s capital structure is relevant. Given firm characteristics, there is only one optimal capital structure. The result is in contrast to the popular interpretation of Modigliani and Miller Proposition 1 that the firm’s financing policy is irrelevant. One of the determinants of optimal financial leverage is the time preferences of the firm’s manager; however, risk preferences do not play any role in determining the optimal capital structure. I find that firms with more impatient managers have proportionally more debt. Prior research suggests that present-biased individuals have significantly higher amounts of credit card debt (Meier and Sprenger, 2010). Cronqvist, Makhija, and Yonker (2012) provide evidence on a positive relation between CEO personal and corporate leverage, which is consistent with behavioral consistency theory. They argue that debt averse CEOs would have less personal debt, and the firms they manage would be less levered, ceteris paribus. Taken together, more patient managers (or managers with a lower subjective discount rate) would have less personal debt, and their firms would have less debt too. The model of this paper supports this prediction. According to the model, the time preferences of the firm’s manager impact the effective discount rate that is used to discount future cash flows to shareholders. Suppose we have two identical firms. Suddenly, the subjective discount rate of the manager of the first firm increases leading to a lower share price, ceteris paribus. To maintain the same capital stock, a firm needs to issue debt securities. This leads to the higher leverage. Thus, a firm whose manager has a higher subjective discount rate is more levered. Static models

³These features will be disabled in certain parts of the analysis.
by construction do not include the time preferences of the firm’s manager and cannot reveal
the important relation between the latter and the firm’s optimal financing policy. Further,
consistent with Modigliani-Miller proposition 1, I show that WACC and the value of the
firm are independent of its capital structure.

In an environment with taxes but without bankruptcy costs, the optimal capital struc-
ture is the mix of equity and debt rather than 100% debt. The results imply that the impact
of taxes on the firm’s optimal capital structure is less important than previously thought.
Firm value and WACC are not impacted by the time preferences of the firm’s manager
and are constant when leverage changes. The results are not consistent with Modigliani
and Miller (1958, 1963) who argue that firm value increases and WACC decreases with
leverage. The results are driven by the capital stock. I show that it does not depend on
the tax rate in this environment. Thus, constant value of assets leads to constant firm
value in order to satisfy the balance sheet equation. This result gives us constant WACC,
assuming fixed EBIT and discounted cash flow at WACC valuation model.

At last, I assume that there are bankruptcy costs but there are no taxes. I show that
the optimal capital stock decreases with the subjective discount rate of the firm’s manager
in this environment. This leads to the lower firm value as the latter is proportional to
capital stock. Thus, there is a negative relation between firm value and financial leverage
(as debt financing increases with the subjective discount rate). Fixed EBIT and lower firm
value imply a positive relation between financial leverage and WACC. The results are in
contrast to Modigliani and Miller (1958) who argue that Modigliani and Miller Proposition
1 is unaffected in this environment. However, the relation between return on equity capital
and debt-to-equity ratio is expressed by a concave function, consistent with Modigliani and
Miller (1958).

To conclude, the study improves our understanding of a firm’s optimal financing choices
in different environments. The results are generally consistent with Modigliani and Miller
Propositions 1 and 2 in the environment without taxes and bankruptcy costs. However, the
first proposition should be presented and interpreted more carefully. A relaxation of the assumptions about either taxes or bankruptcy costs leads to conclusions that are different from those in Modigliani and Miller (1958) with the exception of the non-linear relation between return on equity capital and debt-to-equity ratio when there are bankruptcy costs. I show that the conflicting results are due to the two shortcomings of static models, namely the absence of productive capital stock and the time preferences of the firm’s manager.

This paper is not the first to re-examine or criticize Modigliani and Miller’s propositions. Solomon (1963) argues that the average cost of debt increases with leverage; therefore, the firm’s capital structure is one of the determinants of the cost of capital. However, he does not show this numerically. Brigham and Gordon (1968) make two important assumptions: first, that the corporation’s returns on assets and investment are greater than the rate at which shareholders discount dividends of the unlevered firm; second, that the book value of equity per share does not depend on the debt per share. They find that market value of equity per share is positively impacted by the debt per share. The model of this paper implies that the time preferences of the firm’s manager impact share price and debt per share differently. Share price rises with the subjective discount factor of the firm’s manager; however, debt per share decreases with the subjective discount factor. This leads to a negative relation between share price and debt per share, which is in contrast to the conclusion of Brigham and Gordon (1968). Stiglitz (1969) finds that the conclusions of Modigliani and Miller (1958) still hold if capital markets are not competitive. He shows that only the following two assumptions are necessary for the Modigliani and Miller’s theory to hold: a) individual investors can borrow at the same interest rate as firms, b) there is no bankruptcy. Miller (1977) argues that optimal capital structure for any individual firm does not exist if corporate debt is riskless and if there are both corporate and personal income taxes. The result is driven by the clientele effect. DeAngelo and Masulis (1980) broaden the analysis of Miller (1977) by including non-debt corporate tax shields such as depreciation. They show that the presence of tax shields determines the optimal capital structure for an individual firm.
A number of previous studies analyze the optimal capital structure of a firm using a dynamic model. However, they focus on other issues rather than on how taxes and bankruptcy costs impact the firm’s value and optimal financing policy. Further, the previous studies do not explicitly contrast their results with those in Modigliani and Miller (1958). Below, I briefly discuss the selected research papers that develop and use dynamic models to analyze the dynamics of capital structure. Berens and Cuny (1995) find that the net shield carryforward impacts both the optimal debt and the firm value. Leland (1998) argues that the firm’s optimal leverage increases with hedging activities. Hennessy and Whited (2005) present a dynamic trade-off model and show that there is no target leverage ratio. Titman and Tsyplakov (2007) develop a continuous time model and show that leverage target is impacted by the conflicts of interest between shareholders and debtholders as well as financial distress costs. The model in He (2011) implies a negative relation between pay-for-performance sensitivity and firm size and a positive relation between firm size and optimal debt level. In all the papers above, earnings or firm’s assets, or payout stream are exogenously determined. In this study, these variables are endogenous implying that they depend on the decisions of the firm’s manager. Thus, this paper analyzes the firm’s optimal financing policy from the different perspective and offers new insights on the determinants of the optimal leverage and the firm value.

2 The model

I use a non-stochastic (deterministic) version of the dynamic partial equilibrium model developed in Karpavičius (2014). The model replicates the life and behavior of a representative firm in a dynamic world. I consider a firm with an infinite life span in discrete time. It is assumed that a firm’s manager has rational expectations about the future and

\footnote{In this section, the description of the model is broadly similar to one in Karpavičius (2014). The model does not include any shock as only steady state environment is analyzed in the paper. The reader could simply assume that model includes productivity, demand, or interest rate shocks. However, all shocks would be eliminated in the steady state environment.}
acts completely in the best interests of shareholders. In each time period, the firm’s manager has to choose how much capital to raise in the external equity and debt markets, how much to produce, and how much to invest in capital stock (i.e., fixed assets used in production). The decisions made in the current period will impact firm performance not only in the current period but also in the future. The firm’s manager takes this issue into account. For simplicity, the model is stationary and there is no growth.

A firm produces a single tradable final good that is sold in a competitive market.

2.1 A firm

A standard utility function includes at least one component – consumption. To keep the model as simple as possible, one can assume that the utility function of the firm’s manager depends on shareholder value rather than on her consumption level. In recent years, the equity component (restricted stock and options) of the median CEO of all the ExecuComp firms is equal to around half of the total CEO pay (Murphy, 2013). Thus, a CEO who wants to maximize her utility function that is only determined by her consumption should increase her consumption expenditure and thus her income. Since income is positively impacted by shareholder value (via restricted stock and options), the manager’s ultimate goal is to maximize shareholder value. This would lead to the maximum consumption level for the firm’s manager. Thus, without loss of generality, I assume that the firm’s manager acts in the best interests of current shareholders and maximizes a certain objective function that depends positively on shareholder value.

The firm’s manager has the following intertemporal objective function:

$$E_0 \left( \sum_{t=0}^{\infty} \beta^t U_t \right),$$

(1)

where $\beta$ is the subjective discount factor and reflects the time preferences of the firm’s

The results will not be qualitatively affected if growth is introduced.
manager. The instantaneous objective function, $U_t$, is

$$U_t = \frac{(P_t N_t)^{1-\sigma}}{1-\sigma},$$  \hspace{1cm} (2)$$

where $P_t$ is value of equity per share at time $t$. Both firm’s manager and investors agree on the stock price. $N_t$ is the number of shares outstanding. $\sigma$ is the coefficient of constant relative risk aversion (the inverse of elasticity of substitution). 

The firm’s manager maximizes the objective function subject to the evolution of total equity and asset composition of the firm:

$$P_t N_t = P_{t-1} N_{t-1} + P_{t-1} N_{t-1} - d_t N_{t-1} + \pi_t$$  \hspace{1cm} (3)$$

$$K_t = \kappa(D_t + P_t N_t).$$  \hspace{1cm} (4)$$

Dividends at time $t$, $d_t$, are paid to those who owned shares at time $(t-1)$. Thus, investors who purchase shares at time $t$ are not entitled to receive dividends in this period. $\pi_t$ represents net income. $D_{t-1}$ is debt a firm pays back in period $t$; thus, $D_t$ is new borrowing. For simplicity, it is assumed that debt consists of one-period securities. $K_t$ is capital stock at time $t$. Equation (4) implies that a firm can invest only the $\kappa$ fraction of its financial assets into capital stock. This assumption is introduced in order to make the model more realistic. For example, a mean (median) of fixed assets-to-total assets ratio for all Compustat firms was 0.246 (0.160) in 2009. The rest of the financial capital, $(1-\kappa) \times (D_t + P_t N_t)$, can be seen as working capital. Stock of physical capital, $K_t$, evolves according to:

$$K_t = (1-\delta)K_{t-1} + I_t,$$  \hspace{1cm} (5)$$

\footnote{The model implies that the stock price is equal to the present value of future dividends (see Karpavičius (2014) and Equation B.4 in Appendix B.1 for more details).}

\footnote{The stochastic dynamic partial equilibrium model in Karpavičius (2014) features an exogenous process that represents disagreements between investors and firm’s manager on stock price. In this paper, I do not run any simulations; therefore, the model does not include any stochastic process.}

\footnote{Despite time and risk being interrelated, Andreoni and Sprenger (2012) argue that time preferences are distinct from risk preferences.}
where $\delta$ is the capital depreciation rate. $I_t$ stands for investment.

Firm’s EBIT and net income are given by:

$$
\text{EBIT}_t = S_t - C_t - \delta K_{t-1},
$$

(6)

$$
\pi_t = (S_t - C_t - \delta K_{t-1} - D_{t-1} r_{t-1}) \times (1 - \tau),
$$

(7)

where $S_t$ is sales revenue. $C_t$ is an amount of production input (for example, labor and raw materials). It is assumed that the unit cost of $C_t$ is one. $\tau$ is corporate income tax rate. $r_t$ is the interest rate on debt obtained in time $t$, $D_t$. The interest rate at which a firm can borrow funds depends on its leverage and evolves according to the following equation:

$$
r_t = r^* \left( 1 + \Phi_r \frac{D_t}{D_t + P_t N_t} \right),
$$

(8)

where $r^*$ is a constant and equal to the hypothetical interest rate on corporate bonds for firms with zero leverage. The last term in Equation (8) is the risk premium related to a firm’s financial leverage. $\Phi_r > 0$ is the parameter of risk premium. The definition of interest rate implies that it is an increasing function of a firm’s financial leverage. In the model, debt has advantages (such as tax deductibility of interest expenses and lower costs) and disadvantages (increased bankruptcy risk).

Sales revenue, $S_t$, is the product of output volume, $Y_t$, and the price per output unit, $p_t$:

$$
S_t = Y_t p_t.
$$

(9)

The price per output unit depends on demand for a firm’s products and is given by the following equation:

$$
p_t = \bar{p} \left( \frac{\bar{Y}}{Y_t} \right)^\eta,
$$

(10)

where $\bar{Y}$ and $\bar{p}$ are the demand for a firm’s products and market price in the steady state,
respectively. Parameter $\eta$ is price elasticity of demand.

To produce a single tradable good, a firm uses the following Cobb-Douglas technology:

$$Y_t = K_t^{\alpha}C_t^{1-\alpha},$$  \hspace{1cm} (11)

where $\alpha$ is capital share. Equation (11) implies that production output is the increasing function of capital stock and other production inputs.

As in Modigliani and Miller (1958, 1963), I assume that dividends are exogenous from the perspective of the firm’s manager. Dividends per share, $d_t$, consist of constant and variable parts. The constant part is equal to the steady-state dividends per share, $\bar{d}$, multiplied by $\psi$.\footnote{The term “steady state” refers to the deterministic steady state. Throughout this paper, variables with bars denote steady-state values.} It is equivalent to a certain amount of cash per share distributed to shareholders at the end of each period. The variable part of dividends per share is net income per share multiplied by its weight, $(1 - \psi)$:

$$d_t = \psi \bar{d} + (1 - \psi) \frac{\pi_t}{N_{t-1}},$$  \hspace{1cm} (12)

where $\psi$ is the weighting parameter. In the steady state, total dividends are equal to net income, implying 100% payout policy as in Modigliani and Miller (1958, 1963). The value of the weight of the constant part of dividends, $\psi$, does not impact steady-state dividends per share.\footnote{The steady-state dividends, $\bar{d}$, can be seen as the long-term historical average dividends per share or the target dividends per share. In good times, actual dividends per share exceed the steady-state value but in bad times, actual dividends per share are lower than it. However, the average actual dividends per share are equal to the steady-state value.} Thus, any differences in the results of this paper and in those in Modigliani and Miller (1958, 1963) are not due to assumptions of the payout policy.\footnote{However, $\psi$ impacts dividends per share in a stochastic environment (see Karpavičius 2014).}
2.2 The equilibrium

In each period, the firm’s manager chooses strategy \( \{C_t, K_t, N_t, D_t\}_{t=0}^{\infty} \) to maximize her expected lifetime utility subject to constraints (Equations (3) and (4)), initial values of debt, capital stock, share price, the number of shares outstanding and a no-Ponzi scheme constraint of the form:

\[
\lim_{j \to \infty} \mathbb{E}_t \frac{D_{t+j}}{\prod_{i=1}^{j}(1 + r_i)} \leq 0.
\] (13)

Following Modigliani and Miller (1958), I assume that the firm’s manager does not foresee the possibility that a firm might default. The assumption helps simplify the policy functions and if relaxed, it will not affect the conclusions of the paper.

Maximization of objective function (Equation (1)) subject to the evolution of shareholder value and asset composition of a firm (Equations (3) and (4)) yields the following first-order conditions:

\[
\frac{\partial}{\partial C_t} : C_t = (1 - \alpha) \times (1 - \eta) S_t,
\] (14)

\[
\frac{\partial}{\partial K_t} : \left( \frac{K_t}{\kappa} - D_t \right)^{-\sigma} - \lambda_t + \beta \mathbb{E}_t \left\{ \lambda_{t+1} \left[ \frac{N_{t+1}}{N_t} \right] \right. \\
+ \kappa \psi (1 - \tau) \times \left[ \alpha(1 - \eta) S_t \frac{S_{t+1}}{K_{t+1}} - \delta + \Phi_r r^* \kappa \left( \frac{D_t}{K_t} \right)^2 \right] \right\} = 0,
\] (15)

\[
\frac{\partial}{\partial N_t} : \lambda_t \left[ \frac{1}{N_{t-1}} \left( \frac{K_{t-1}}{\kappa} - D_{t-1} \right) \right] = \beta \mathbb{E}_t \left\{ \lambda_{t+1} \left[ \frac{N_{t+1}}{N_t^2} \left( \frac{K_t}{\kappa} - D_t \right) + \psi d \right] \right\},
\] (16)

\[
\frac{\partial}{\partial D_t} : - \left( \frac{K_t}{\kappa} - D_t \right)^{-\sigma} + \lambda_t = \beta \mathbb{E}_t \left\{ \lambda_{t+1} \left[ \frac{N_{t+1}}{N_t} + \psi [1 - \tau] r^* \left[ 1 + 2 \Phi_r \kappa \frac{D_t}{K_t} \right] \right] \right\},
\] (17)

where \( \lambda_t \) is a Lagrange multiplier. Equation (14) defines the optimal level of production input, \( C_t \), and Equations (15)-(17) are Euler conditions.
The equilibrium of the model is defined by the evolution of shareholder value, asset composition constraint, first-order conditions, and several variable definitions (in total 14 equations). The number of endogenous variables is equal to the number of equations; thus, the model can be solved. I analyze the properties of the model in the steady state. It would help to understand long-term equilibrium relations among the model’s variables. To do so, I solve for the non-stochastic steady state of the model by using the following procedure: the time subscripts are dropped and the steady-state values of each endogenous variable are expressed in terms of parameters.

When the time subscripts are dropped, the model reduces to 12 equations: Equation (10) cancels out and the steady-state expressions of Equations (3) and (12) are identical. To express the steady-state values of each endogenous variable in terms of parameters and constants, the number of endogenous variables must be equal to the number of equations. Thus, I assume that steady-state values of the number of shares outstanding, \( \bar{N} \), and EBIT are known.

### 2.3 Calibration

The model is calibrated partially as in Karpavičius (2014). It is assumed that the variables are measured quarterly. The calibration of the model is summarized in Table 1. I normalize the steady-state value for the number of shares outstanding, \( \bar{N} \), to one. Following Modigliani and Miller (1958), I assume that EBIT are constant. They are set to one in the steady state. I assume that the model is stationary. This implies that capital stock, \( \bar{K} \), is constant in the steady state and that depreciation, \( \bar{K}\delta \), is equal to investments, \( \bar{I} \). Thus, free cash flow to the firm (FCFF) and EBIT are equivalent in the steady state if there are no taxes.

Quarterly discount factor, \( \beta \), is set to 0.98. The coefficient of manager’s risk aversion,
Table 1: Calibration of the parameters
This table presents the calibrated parameter values.

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{N}$</td>
<td>Shares outstanding in the steady state</td>
<td>1</td>
</tr>
<tr>
<td>EBIT</td>
<td>EBIT in the steady state</td>
<td>1</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Fixed assets-to-capital ratio</td>
<td>0.28</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Corporate income tax rate</td>
<td>0.3</td>
</tr>
<tr>
<td>$r^*$</td>
<td>Interest rate for unlevered firm</td>
<td>0.01</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Capital share</td>
<td>0.3</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Price elasticity of demand</td>
<td>0.2</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Subjective discount factor</td>
<td>0.98</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Weight of the constant part of dividends</td>
<td>0.8</td>
</tr>
<tr>
<td>$\Phi_r$</td>
<td>Parameter of risk premium</td>
<td>1</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Capital depreciation rate</td>
<td>0.025</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Coefficient of constant relative risk aversion</td>
<td>2</td>
</tr>
</tbody>
</table>

$\sigma$, is two.
I assume that $\psi$ is 0.8. It implies that the weight of constant dividend is 0.8. [1] Modigliani and Miller (1958, 1963) assume 100% payout. In this paper, the payout depends on parameter $\psi$ in the stochastic environment; however, the payout is invariant with respect to $\psi$ in the steady state. The whole net income is distributed to shareholders in the latter case as in [Modigliani and Miller (1958, 1963)]. Following macroeconomic literature, quarterly capital depreciation rate, $\delta$, is set to 0.025 and capital share in the production function, $\alpha$, is equal to 0.3. Fixed assets-to-net assets ratio is equal to 0.285 for the population of COMPUSTAT firms during 1980-2009; therefore, $\kappa$ is set to 0.28.

The quarterly interest rate on corporate bonds for unlevered firm, $r^*$, is set to 0.01. It implies that the hypothetical annual interest rate for firms without debt is 4%. The corporate income tax rate, $\tau$, is 0.3, which is approximately equal to an average value of corporate marginal tax rate simulated in [Graham and Mills (2008)]. I assume that price elasticity of demand, $\eta$, is 0.2. It implies that if the production supply increases by 10%, the sale price decreases by 2%, and vice versa. The parameter of risk premium, $\Phi_r$, is set to one. It implies that if a firm’s leverage increases by one percentage point, the quarterly

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1. Murphy (1999) uses one, two, and three as a value of relative risk aversion. I show that different values of $\sigma$ do not affect the results.
interest rate will increase by 1 basis points if \( r^* \) is 0.01.

The calibrated parameter values imply that key firm characteristics in the steady state are similar to those of the population of COMPUSTAT firms during 1980-2009. For example, debt-to-assets ratio is 0.215 whereas book and market leverage ratios for COMPUSTAT firms are 0.282 and 0.185, respectively. The steady-state value of tangibility (fixed assets, \( \bar{K} \), over total assets, \( \bar{D} + \bar{P}^b \bar{N} \)) is 0.28 and it is close to empirical value (0.285). Real quarterly return on CRSP value weighted index is 0.022 for the period 1980-2009. The calibration implies slightly higher return (0.026)\(^{15}\). Thus, from the firm’s perspective, equity financing is more expensive than debt financing.

3 Results

In this section, I analyze the implications of the model on leverage, firm value, and WACC and compare them with the conclusions of Modigliani and Miller (1958).

The steady-state firm value, \( \bar{V} \), is defined as:

\[
\bar{V} = \bar{P} \bar{N} + \bar{D}. \tag{18}
\]

Taking into account, the asset composition of the firm (Equation (4)), firm value can be also expressed as:

\[
\bar{V} = \frac{\bar{K}}{\kappa}. \tag{19}
\]

Equation (19) implies that the value of the firm is proportional to capital stock. Thus, \( \bar{V} \) stands for the optimal firm size. As shareholders care about the return on their investment rather than about firm size, implications on \( \bar{V} \) are less important than previously thought and covered in the literature. In this paper, a firm uses two inputs – \( K_{t-1} \) and \( C_t \) – to produce a tradable final good. Thus, there are an infinite number of different combinations

\(^{15}\)Table A.1 in Appendix A presents the steady-state values of all variables.
of $K_{t-1}$ and $C_t$ that generate the same EBIT. Since firm value is proportional only to capital stock, greater firm value is not equivalent to higher return to shareholders. Instead, it might simply indicate that the firm uses less $C_t$ in the production to generate the same EBIT. Further, Equation (19) shows that if capital stock is independent of a parameter then the value of the firm will be also unaffected by it.

Weighted average cost of capital, WACC, is defined as follows:

$$WACC = \frac{\bar{D}}{\bar{D} + \bar{P}N} \bar{r}(1 - \tau) + \left(1 - \frac{\bar{D}}{\bar{D} + \bar{P}N}\right) \bar{r}^e,$$  \hspace{1cm} (20)

where $\bar{r}^e$ is dividend yield in the steady state$^{16}$

$$\bar{r}^e = \frac{\bar{d}}{\bar{P}} = \frac{1 - \beta}{\psi \beta}.$$  \hspace{1cm} (21)

Thus, dividend yield that is equivalent to the return on shareholder capital is affected only by two factors: the time preference and dividend smoothness parameters. $\bar{r}^e$ is impacted by neither tax rate nor bankruptcy costs, $\Phi_r$, implied by the amount of debt, $D_t$.

To emphasize the critical impact of bankruptcy costs, I initially analyze the firm’s optimal financing policy, assuming that borrowing costs do not depend on leverage. Later, I relax this assumption and analyze whether it affects the previous conclusions. I find that the impact of taxes is rather marginal; thus, I present the results for different tax regimes in the same sections.

$^{16}$See Appendix B.1 for more details.
3.1 Optimal capital structure, WACC, and firm value when there are no bankruptcy costs

If $\Phi_r = 0$, leverage, $L$, WACC, and firm value, $V$, in the steady state are expressed as follows:[17]

\[
\bar{L} = \frac{\bar{D}}{\bar{D} + PN} = 1 - \frac{\kappa \beta \psi \bar{d} \bar{N}}{K(1 - \beta)}, \text{ where}
\]

\[
\bar{d} = \frac{\text{EBIT} - r^*K}{1 - \tau - \frac{\beta \psi \eta r^*}{1 - \beta}} \quad \text{and}
\]

\[
\bar{K} = \frac{\alpha(1 - \eta)\text{EBIT}}{r^* \eta + \alpha(1 - \eta)} + \delta \eta,
\]

\[
\text{WACC} = \frac{(1 - \tau)\text{EBIT}}{\bar{K}} = \frac{(1 - \tau)\text{EBIT}}{\frac{\alpha(1 - \eta)\text{EBIT}}{r^* \eta + \alpha(1 - \eta)} + \delta \eta \kappa} = \frac{(1 - \tau)\text{EBIT}}{\bar{V}},
\]

\[
\bar{V} = \frac{\bar{K}}{\kappa} = \frac{\alpha(1 - \eta)\text{EBIT}}{r^* \eta + \alpha(1 - \eta)} + \delta \eta \kappa.
\]

Equation (22) shows the optimal capital structure of a firm given its characteristics.

Equation (22) has several important implications. First of all, a firm’s capital structure is relevant, in contrast to the popular interpretation of Modigliani and Miller Proposition 1. Given firm characteristics, there is only one optimal capital structure. The optimal capital structure always exists unless the denominators of Equations (22)-(24) are zeros. There is no optimal capital structure when $\beta = 1$, $\tau = 1$, $\kappa = 0$, $\alpha = 0$, $\eta = 1$, $\text{EBIT} = 0$ as well as in other cases when the denominators of Equations (23) and (24) are equal to zero. The first condition $\beta = 1$ means that the firm’s manager has no time preferences and her time value of money is zero. The second condition $\tau = 1$ means that the tax rate is 100%. Thus, the firm’s net income is always zero. The condition $\kappa = 0$ means that a firm has no productive capital and firm’s output is zero (refer to Equations (4) and (11)).

[17] See Appendices B.2-B.5 for more details.
\( \alpha = 0 \) implies that capital stock is useless in the production. Thus, a firm should have no productive capital. \( \eta = 1 \) condition implies constant sales revenues (see Equations (9) and (10)). \( \text{EBIT} = 0 \) implies zero sales in equilibrium. In all cases above, the firm’s manager cannot improve shareholder value regardless of her decisions and efforts. Thus, capital structure is indeed irrelevant. The conditions are quite unrealistic and can be ignored.\(^{18}\)

Second, Equations (22) and (23) reveal the importance of the time preferences of the firm’s manager. Leverage ratio depends on the manager’s subjective discount factor, \( \beta \). Prior empirical research suggests that present-biased individuals have significantly higher amounts of credit card debt (see Meier and Sprenger, 2010) and that CEO personal leverage and the leverage of the firm she manages are positively related (Cronqvist, Makhija, and Yonker, 2012). Taken together, this implies that firms whose managers have lower subjective discount rate would be less levered. Equations (22)-(24) show that the model of this paper supports this prediction. Static models by construction do not include the time preferences of the firm’s manager and cannot reveal the important relation between the latter and the firm’s optimal financing policy. Thus, Equations (22)-(24) show the advantages of using dynamic models.

Third, the results imply that in the presence of taxes, an optimal capital structure is still a certain mix of debt and equity. I show that risk preferences do not play any role in determining the optimal capital structure. The results hold regardless of whether the firm’s manager is risk averse, risk neutral, or risk seeking. Further, Equation (22) shows that optimal capital structure depends on payout policy. Financial leverage is lower for firms with smoother dividends.

\(^{18}\)It is difficult to develop hypothetical cases where these conditions are realistic. One possible example is when a firm is in default. In this case, the value of equity is zero regardless of the firm’s debt level. Thus, firm’s capital structure is irrelevant. Another example is the case when the value of the firm’s assets is zero \( (K = 0) \). Then, according to Equation (1), the total value of equity is equal to the negative debt. Shareholder equity is non-negative. Thus, this would suggest that such a firm is like a bank, in that it produces nothing and has no operating cash flows. A firm collects cash from shareholders (as shareholder equity) and then lends it to others (thus, the debt is negative). Shareholders would receive dividends that are equal to after-tax interest income. However, even here the capital structure is not irrelevant. The firm’s leverage is equal to \( -0.5 \) as the sum of debt and equity must be zero.
To analyze how the time preferences of the firm’s manager impact firm’s leverage, I compute the first- and second-order partial derivatives of debt ratio with respect to the time preferences of the firm’s manager. To simplify the task, I introduce a subjective discount rate, $R$:

$$R = \frac{1 - \beta}{\beta}. \quad (27)$$

Further, I express financial leverage ratio (Equation (22)) in terms of subjective discount rate:

$$\bar{L} = 1 - \kappa \psi \bar{d} \bar{N} \overline{K} R,$$

where

$$\bar{d} = \frac{\overline{EBIT} - r^* \bar{K}}{\overline{N} \overline{K} - \psi \bar{N} \overline{r} \overline{K}} \quad \text{and} \quad \bar{K} = \frac{\alpha (1 - \eta) \overline{EBIT}}{r^* \kappa [\eta + \alpha (1 - \eta)] + \delta \eta}. \quad (28)$$

The first- and second-order partial derivatives of debt ratio with respect to a subjective discount rate are as follows:

$$\frac{\partial \bar{L}}{\partial R} = \psi (1 - \tau) \left( \frac{\kappa \overline{EBIT} \bar{K}}{\overline{K} R} - r^* \right) > 0 \quad (30)$$

$$\frac{\partial^2 \bar{L}}{\partial R^2} = -2 \psi (1 - \tau) \left( \frac{\kappa \overline{EBIT} \bar{K}}{\overline{K} R} - r^* \right)^2 < 0 \quad \text{where} \quad (31)$$

$$\bar{K} = \frac{\alpha (1 - \eta) \overline{EBIT}}{r^* \kappa [\eta + \alpha (1 - \eta)] + \delta \eta}. \quad (31)$$

$\frac{\kappa \overline{EBIT}}{\overline{K}}$ is equivalent to $\frac{\overline{EBIT}}{\overline{V}}$ (see Equation (19)) and measures the operating profitability of the firm’s assets. It generally exceeds the hypothetical interest rate on the unlevered firm’s debt. Thus, $\left( \frac{\kappa \overline{EBIT}}{\overline{K}} - r^* \right) > 0$. Normally, a subjective discount rate is greater than $r^*$; therefore, the denominator of Equation (31) is positive regardless of $\tau$ and $\psi$. This implies that the first-order partial derivative is positive and the second-order partial derivative is negative, meaning that financial leverage increases with a subjective discount.
rate but at decreasing rate.

Further, I compute the cost of capital and firm value for different values of leverage. Since the debt policy is endogenous in the model, the analysis of the impact of leverage on the variables is not straightforward. The analysis consists of two steps. First, one needs to choose the parameter which value will be changed in order to achieve different values of leverage. All parameters in Equation (28) but one are related to industry or economy. Only $R$ is a firm-specific variable and reflects the time preferences of the firm’s manager.\textsuperscript{19} The determinants of the manager’s time preferences include her age, life experience, wealth, and the design of the executive remuneration contract. For example, Bhagat, Bolton, and Subramanian (2011) and Mehran (1992) report the significant relation between CEO compensation structure and firm’s leverage.\textsuperscript{20} This makes $R$ highly suitable for the analysis. Then I analyze the relation between the subjective discount rate and either WACC or firm value.

Equations (25) and (26) indicate that WACC and firm value do not depend on $R$. Thus, the first- and second-order partial derivatives of WACC and firm value with respect to a subjective discount rate are zeros. Consistent with Modigliani-Miller proposition 1, the results suggest that there is no relation between leverage and either WACC or firm value.

Modigliani and Miller Proposition 2 says that: “the expected yield of a share of stock is equal to the appropriate capitalization rate $\rho_k$ for a pure equity stream in the class, plus a premium related to financial risk equal to the debt-to-equity ratio times the spread between $\rho_k$ and $r$” (Modigliani and Miller 1958, p. 271).\textsuperscript{21} It is derived from Modigliani and Miller Proposition 1 that holds in a dynamic setting; therefore, Proposition 2 also holds.

\textsuperscript{19}One might argue that $\psi$ is also a firm-specific parameter. Unreported analysis using $\psi$ instead of $R$ leads to qualitatively similar results. According to Karpavičius (2014), smaller $\psi$ is related to the greater risk to the shareholders; therefore, the results in this paper hold even when Modigliani and Miller’s (1958) original assumption of risk classes is violated and are consistent with those in Stiglitz (1969).

\textsuperscript{20}Similarly, May (1995) argues that CEO compensation structure impacts CEO’s incentives and firm’s risk management decisions. May (1995) finds that firms with greater managerial ownership tend to make diversifying acquisitions.

\textsuperscript{21}$r$ denotes the rate of return on bonds in Modigliani and Miller (1958).
The results are generally consistent with Modigliani and Miller Propositions 1 and 2; however, the first proposition should be presented and interpreted more carefully. The results of this paper imply that, given certain firm characteristics, there is only one optimal capital structure. Thus, the firm’s financing policy is not irrelevant even if there are no taxes and bankruptcy costs. Any changes in the time preferences of the firm’s manager would lead to a new optimal capital structure. However, the value of the firm and WACC would remain unaltered because they are determined by the capital stock which is independent of the time preferences of the firm’s manager.

Panel A in Table 2 reports key static results for different values of subjective discount rate, \( R \). The model is able to produce the positive risk-return relation. We observe the positive relation between dividend yield (return on equity capital) and financial leverage (risk for equity holders). As discussed above, firm value, \( \bar{V} \), and WACC are independent of \( R \).

In the economy with taxes (0 < \( \tau < 1 \)) but without bankruptcy costs (\( \Phi_r = 0 \)), the results and conclusions are similar to those discussed above. Given certain firm characteristics, there is only one optimal capital structure. However, it changes with the time preferences of the firm’s manager. Firm value and WACC are not impacted by the time preferences and are constant when leverage changes. The results are not consistent with Modigliani and Miller (1958, 1963) who argue that firm value increases and WACC decreases with leverage. The firm value is constant for different values of the time preference parameter because taxes impact both production factors (\( C_t \) and \( K_{t-1} \)); thus, the \( \bar{C}/\bar{K} \) ratio is unchanged.\(^{22}\) This leads to the constant capital stock, \( \bar{K} \), and firm value, \( \bar{V} \). Given constant EBIT and firm value, WACC is also constant (see Equation (25)).

Panel B in Table 2 reports key static results for different values of subjective discount rate, \( R \), if the tax rate is 30%. The values of all variables, except for financial leverage, \( \bar{L} \), and WACC are identical to those in Panel A. Taxes reduce effective borrowing costs;\(^{22}\)

\(^{22}\)Only Equation (17) treats capital stock differently in the presence of taxes; however, the discriminatory treatment vanishes if \( \Phi_r = 0 \).
Table 2: The impact of subjective discount rate on selected variables

This table presents the impact of subjective discount rate, \( R = \frac{1-\beta}{\beta} \), on selected variables. Panels A, B, C, and D show the results for different values of \( \tau \) and \( \Phi_r \). \( \bar{L} = \frac{\bar{D}r}{\bar{D} + \bar{P}N} \) is financial leverage in the steady state. \( \bar{r}^e = \frac{d}{\bar{P}} \) is dividend yield in the steady state. \( \bar{r} \) is interest rate in the steady state. \( \bar{C} \) is an amount of production input in the steady state. \( \bar{S} \) is sales revenue in the steady state. \( \bar{K} \) is capital stock in the steady state. \( \bar{V} \) is firm value in the steady state. WACC = \( \bar{L}\bar{r}(1-\tau) + (1-\bar{L})\bar{r}^e \) is the weighted average cost of capital.

| \( R \)  | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 |
| \( \beta \) | 0.990 | 0.9804 | 0.971 | 0.962 | 0.952 |

Panel A. \( \tau = 0 \) and \( \Phi_r = 0 \)

| \( \bar{L} \) | -4.667 | 0.056 | 0.485 | 0.646 | 0.730 |
| \( \bar{r}^e \) | 0.013 | 0.025 | 0.038 | 0.050 | 0.063 |
| \( \bar{r} \) | 0.010 | 0.010 | 0.010 | 0.010 | 0.010 |
| \( \bar{C}/\bar{S} \) | 0.560 | 0.560 | 0.560 | 0.560 | 0.560 |
| \( \bar{S}/\bar{K} \) | 0.253 | 0.253 | 0.253 | 0.253 | 0.253 |
| \( \bar{C}/\bar{K} \) | 0.142 | 0.142 | 0.142 | 0.142 | 0.142 |
| \( \bar{V} \) | 41.379 | 41.379 | 41.379 | 41.379 | 41.379 |
| WACC | 0.024 | 0.024 | 0.024 | 0.024 | 0.024 |

Panel B. \( \tau = 0.30 \) and \( \Phi_r = 0 \)

| \( \bar{L} \) | -0.803 | 0.449 | 0.675 | 0.769 | 0.821 |
| \( \bar{r}^e \) | 0.013 | 0.025 | 0.038 | 0.050 | 0.063 |
| \( \bar{r} \) | 0.010 | 0.010 | 0.010 | 0.010 | 0.010 |
| \( \bar{C}/\bar{S} \) | 0.560 | 0.560 | 0.560 | 0.560 | 0.560 |
| \( \bar{S}/\bar{K} \) | 0.253 | 0.253 | 0.253 | 0.253 | 0.253 |
| \( \bar{C}/\bar{K} \) | 0.142 | 0.142 | 0.142 | 0.142 | 0.142 |
| \( \bar{V} \) | 41.379 | 41.379 | 41.379 | 41.379 | 41.379 |
| WACC | 0.017 | 0.017 | 0.017 | 0.017 | 0.017 |

Panel C. \( \tau = 0 \) and \( \Phi_r = 1 \)

| \( \bar{L} \) | -0.252 | 0.016 | 0.231 | 0.394 | 0.514 |
| \( \bar{r}^e \) | 0.013 | 0.025 | 0.038 | 0.050 | 0.063 |
| \( \bar{r} \) | 0.007 | 0.010 | 0.012 | 0.014 | 0.015 |
| \( \bar{C}/\bar{S} \) | 0.560 | 0.560 | 0.560 | 0.560 | 0.560 |
| \( \bar{S}/\bar{K} \) | 0.169 | 0.258 | 0.314 | 0.347 | 0.367 |
| \( \bar{C}/\bar{K} \) | 0.094 | 0.144 | 0.176 | 0.194 | 0.205 |
| \( \bar{V} \) | 72.650 | 40.390 | 31.575 | 27.950 | 26.201 |
| WACC | 0.014 | 0.025 | 0.032 | 0.036 | 0.038 |

Panel D. \( \tau = 0.30 \) and \( \Phi_r = 1 \)

| \( \bar{L} \) | -0.131 | 0.204 | 0.433 | 0.578 | 0.672 |
| \( \bar{r}^e \) | 0.013 | 0.025 | 0.038 | 0.050 | 0.063 |
| \( \bar{r} \) | 0.009 | 0.012 | 0.014 | 0.016 | 0.017 |
| \( \bar{C}/\bar{S} \) | 0.560 | 0.560 | 0.560 | 0.560 | 0.560 |
| \( \bar{S}/\bar{K} \) | 0.211 | 0.308 | 0.354 | 0.375 | 0.386 |
| \( \bar{C}/\bar{K} \) | 0.118 | 0.172 | 0.198 | 0.210 | 0.216 |
| \( \bar{V} \) | 52.476 | 32.379 | 27.325 | 25.483 | 24.675 |
| WACC | 0.013 | 0.022 | 0.026 | 0.027 | 0.028 |
therefore, financial leverage is higher and WACC is smaller in Panel B compared to the results in Panel A. To investigate the impact of taxes, I compare the values of main variables for the similar values of leverage under different tax regimes. In Panels A and B, I identify two columns (they are shaded) where the leverage values are similar across the panels. If $R = 0.03 \ (0.04)$ and $\tau = 0$, leverage is $0.485 \ (0.646)$ and if $R = 0.02 \ (0.03)$ and $\tau = 0.3$, leverage is $0.449 \ (0.675)$. If we consider the first shaded column, leverage is higher in Panel A. However, if we look at the second shaded column, the leverage is higher in Panel B. The differences in leverage values across the two panels are relatively small, allowing us to compare the values of other variables. Firstly, we observe that $\bar{r}, \bar{C}/\bar{S}, \bar{S}/\bar{K}, \bar{C}/\bar{K}, \text{ and } \bar{V}$ do not depend on the tax regime. However, $\bar{r}^e$ and WACC are smaller in Panel B. Taxes reduce net income of the firm. Thus, the return on equity capital, $\bar{r}^e$, is smaller in the presence of taxes for the same level of risk (i.e., financial leverage). Lower $\bar{r}^e$ and after-tax borrowing costs, $(1 - \tau)\bar{r}$, lead to the lower WACC.

Would arbitrage conducted by investors lead to the higher values of levered firms in the presence of taxes as shown in [Modigliani and Miller (1958)]? The answer is no. As the results above indicate, the firm value is equal to its book value. Consider a marginal case where $\kappa = 1$ implying that firms invest all their financial assets in capital stock. What is a value of such firm? The firm is as valuable as its capital stock. In a world with perfect markets and an infinite number of firms and investors, a firm cannot be more valuable than its assets as investors would be able to replicate its earnings by establishing a new firm with identical assets to the former firm. Similarly, if the value of the firm’s assets exceeds the value of the firm, investors can liquidate the firm and sell its assets. Thus, firm value is equal to the value of its assets in equilibrium. This explains why the firm value doesn’t increase with leverage as argued in [Modigliani and Miller (1958)]. Their model does not include capital stock; therefore, they could not relate the market value of the firm to the value of the firm’s assets.
3.2 Optimal capital structure, WACC, and firm value when there are bankruptcy costs

Taxes impact both production factors ($K_{t-1}$ and $C_t$); however, $\Phi_r$ impacts only $K_{t-1}$. $\Phi_r \neq 0$ implies that firm’s borrowing costs rise with leverage. According to the first-order conditions with respect to capital stock and debt (Equations (15) and (17)), bankruptcy costs, $\Phi_r$, impacts capital stock, $K_t$, but not $C_t$ or $S_t$. Any impact on sales revenue would be also on an amount of production input, $C_t$, due to the linear relation between these two variables (see Equation (14)). Further, capital stock is an endogenous state variable, that is an endogenous variable which appears in the previous period. Thus, one could expect that there is a relation between $\bar{K}$ and $\beta$ and that the optimal $\bar{C}/\bar{K}$ ratio is impacted by the subjective discount factor, $\beta$.

If we assume that borrowing costs increases with leverage (i.e., $\Phi_r \neq 0$), then:

$$\bar{L} = 1 - \frac{\kappa \beta \psi \bar{d} \bar{N}}{\bar{K}(1 - \beta)},$$

$$\bar{d} = \frac{(1 - \beta)\bar{K}}{\beta \psi \bar{N}} \times \sqrt{\frac{\delta}{r^* \Phi_r \kappa \times \eta + \alpha(1 - \eta)} + \frac{1 + \Phi_r}{\Phi_r \kappa^2} - \frac{EBIT}{r^* \Phi_r \kappa \bar{K}} \times \frac{\alpha(1 - \eta)}{\eta + \alpha(1 - \eta)}},$$

(32)

$$\bar{V} = \frac{\bar{K}}{\kappa},$$

where $\bar{K}$ depends on ten different parameters (on all parameters in Table 1 except $\bar{N}$ and $\sigma$) and is expressed by a complicated equation (see Appendices B.6 and B.7).

The direction of impacts of the subjective discount rate on leverage, WACC, and firm value are not obvious, due to the complexity of the equations. Thus, instead of deriving first- and second-order partial derivatives of the variables with respect to a subjective discount rate, I compute their values for different values of $R$. Panel C in Table 2 reports key static results for different values of subjective discount rate, $R$, if $\tau = 0$ and $\Phi_r = 1$.

The comparison of the results with those in Panel A shows that firms are generally less levered in the presence of bankruptcy costs as borrowing becomes more expensive. As
expected, $\bar{C}/\bar{K}$ ratio changes with $\mathbb{R}$ (see Panel C in Table 2). The ratio increases with $\mathbb{R}$, implying that firms with more present-biased managers have less capital stock (as $\bar{C}$ does not depend on $\mathbb{R}$). Due to the asset composition (see Equation (11)), firm value is proportional to capital stock. Therefore, such firms are less valuable. As FCFF is the same for all values of $\mathbb{R}$, greater firm value implies smaller WACC. Thus, firm value decreases and WACC increases with the subjective discount rate of the firm’s manager. The results are in contrast to Modigliani and Miller (1958) who argue that Modigliani and Miller Proposition 1 is unaffected if interest rate increases with leverage.

Modigliani and Miller (1958, pp. 274-275) argue that “the relation between common stock yields and leverage will no longer be the strictly linear one given by the original Proposition II. If $r$ increases with leverage, the yield $i$ will still tend to rise as $D/S$ increases, but at a decreasing rather than a constant rate.” Due to the complexity of the expressions of $\bar{d}$ and $\bar{K}$, I do not analyze the relation between return on equity capital, $\bar{r}^{e}$, and debt-to-equity ratio, $\bar{D}/\bar{E}$, analytically by finding the first-order conditions. Instead, I plot $\bar{r}^{e}$ against $\bar{D}/\bar{E}$ when $\Phi_{r} = 0$ and $\Phi_{r} = 1$ (see Figure 1). If $\Phi_{r} = 0$, the relation is expressed by the straight line (correlation coefficient between $\bar{r}^{e}$ and $\bar{D}/\bar{E}$ is one). Thus, Modigliani and Miller Proposition 2 holds in a dynamic setting when the interest rate is constant. If $\Phi_{r} = 1$, the relation is expressed by a concave function as argued by Modigliani and Miller (1958) (correlation coefficient between $\bar{r}^{e}$ and $\bar{D}/\bar{E}$ is equal to 0.993 for $\mathbb{R} = \{0.01, 0.02, 0.03, 0.04, 0.05\}$).

The dotted line shows the linear trend to emphasize the non-linear relation between return on equity capital, $\bar{r}^{e}$, and debt-to-equity ratio, $\bar{D}/\bar{E}$, in Figure 1b. To conclude, the results are consistent in both cases with those in Modigliani and Miller (1958).

If taxes are introduced, the results do not change substantially. As one could expect, firms would become more levered due to the tax shield of debt (see Panel D in Table 2).%24 In Modigliani and Miller (1958), $i$ and $D/S$ denote the expected yield on the stock of a company and debt-to-equity ratio, respectively.

%24The values in the third column of Panel D in Table 2 are slightly different from those implied by calibration as they have been computed assuming that $\beta \approx 0.9804$ rather than $\beta$ is equal to its benchmark value (0.98).
Figure 1: The relation between return on equity capital, $\bar{r}_e$, and debt-to-equity ratio, $\frac{D}{E}$, in the steady state. The dotted line in Figure 1b shows the linear trend.

Shaded columns in Panels C and D in Table 2 isolate cases where firms have similar leverage in both environments. The leverage is higher in the first shaded column when there are no taxes. In the second shaded column, leverage is lower in the environment without taxes. Consistent with the results when there are no bankruptcy costs, all variables except return on equity capital and WACC have identical, or very similar, values across the both panels, indicating that taxes do not affect production decisions. The increase in tax rate from zero to 0.3 leads to a lower return on equity capital and WACC.

If the market value of the firm is not equal to $\bar{V}$, investors can follow the same strategies to generate riskless profit as discussed at the end of the previous section.

4 Conclusion

This paper enhances our understanding of the determinants of the firm’s optimal capital structure and cost of capital in different environments. The study employs a dynamic partial equilibrium model for this exercise.

I show that a firm’s capital structure is relevant regardless of the existence of taxes and bankruptcy costs. Given firm characteristics, there is only one optimal capital structure. In an environment without taxes and bankruptcy costs, firm value and WACC are not
impacted by the time preferences and are constant when leverage changes. The results are consistent with Modigliani and Miller Propositions 1 and 2. However, the first proposition should not be understood as indication that the firm’s financing policy is irrelevant. In fact, there is no such claim in Modigliani and Miller (1958). It appeared in later works written by other scholars. Static models utilized by previous studies by construction are not able to reveal that there is only one optimal capital structure, given firm characteristics. Thus, this paper shows the advantages of using a dynamic model. It is based on more realistic assumptions than static models; thus, it is more complicated. The new insights of the firm’s financing policy I obtain suggest that the additional complexity pays off.

In an environment with taxes but without bankruptcy costs, the optimal capital structure is the mix of equity and debt rather than 100% debt. Firm value and WACC are not impacted by the time preferences and are constant when leverage changes. The results are not consistent with Modigliani and Miller (1958, 1963) who argue that firm value increases and WACC decreases with leverage. Thus, the impact of the taxes on the firm’s optimal financing policy is less important than previously thought.

If there are bankruptcy costs but there are no taxes, there is a positive (negative) relation between financial leverage and WACC (firm value). The results are in contrast to Modigliani and Miller (1958) who argue that Modigliani and Miller Proposition 1 is unaffected in this environment. However, the relation between return on equity capital and debt-to-equity ratio is expressed by a concave function, consistent with Modigliani and Miller (1958).

Table 3 summarizes the results of this study and provides a comparison with those in Modigliani and Miller (1958, 1963). The results of this paper are generally consistent with Modigliani and Miller Propositions 1 and 2 in an environment without taxes and bankruptcy costs. However, the first proposition should be presented and interpreted more carefully. A relaxation of the assumptions about either taxes or bankruptcy costs leads to conclusions that are different from those in Modigliani and Miller (1958) with the
exception of the non-linear relation between return on equity capital and debt-to-equity ratio when there are bankruptcy costs. The conflicting results are due to the shortcomings of static models, namely the absence of productive capital stock and the time preferences of the firm’s manager.

The study shows how the time preferences of the firm’s manager impact the firm’s optimal capital structure and WACC. The increase in the subjective discount factor leads to lower firm’s leverage and WACC in the more realistic environment with taxes and bankruptcy costs. This suggests that the corporate boards and perhaps regulators should pay more attention to the design of CEO compensation package that impacts CEO’s risk and time preferences.

This paper highlights the importance of the time preferences of the firm’s manager. Two recent studies help relate the time preferences of the firm’s manager with corporate leverage. Meier and Sprenger (2010) find that present-biased individuals have significantly higher amounts of credit card debt. Cronqvist, Makhija, and Yonker (2012) argue that debt averse CEOs would have less personal debt and the firms they manage would be less levered. This implies that firms with more patient managers (or firms whose managers have a lower subjective discount rate) would be less levered. The model of this paper supports
these predictions. I show that risk preferences do not play any role in determining the optimal capital structure, WACC, and firm value. The results hold regardless of whether the firm’s manager is risk averse, risk neutral, or risk seeking.

The model of this paper assumes that the subjective discount factor of the firm’s manager is constant. It is a standard assumption used in the majority of neo-classical and post-Keynesian models. In this paper, the subjective discount factor of the firm’s manager plays an important role in explaining why the optimal capital structure always exists. Alternatively, one could assume an endogenous discount factor, such as in Uzawa (1968). From the modeling perspective, the nature of the subjective discount factor does not affect the values of endogenous variables in the steady state.[25] Thus, even if I assumed the endogenous subjective discount factor, the results and conclusions of this paper would still hold.

[25] Kim and Kose (2003) find that the nature of the subjective discount factor (fixed vs. endogenous) has only a marginal impact on the dynamic properties of the real business cycle model.
### A Additional table

Table A.1: Steady-state values

This table presents the values of the variables in the steady state. The model is calibrated as in Table 1.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C$</td>
<td>Amount of production input</td>
<td>1.59</td>
</tr>
<tr>
<td>$D$</td>
<td>Debt</td>
<td>6.90</td>
</tr>
<tr>
<td>$I$</td>
<td>Investment</td>
<td>0.224</td>
</tr>
<tr>
<td>$K$</td>
<td>Capital stock</td>
<td>8.97</td>
</tr>
<tr>
<td>$P$</td>
<td>Value of equity per share</td>
<td>25.14</td>
</tr>
<tr>
<td>$\bar{S}$</td>
<td>Sales revenue</td>
<td>2.78</td>
</tr>
<tr>
<td>$\bar{Y}$</td>
<td>Output volume</td>
<td>2.63</td>
</tr>
<tr>
<td>$\bar{d}$</td>
<td>Dividends in the steady state</td>
<td>0.641</td>
</tr>
<tr>
<td>$\bar{p}$</td>
<td>Price per output unit</td>
<td>1.06</td>
</tr>
<tr>
<td>$\bar{r}$</td>
<td>Interest rate</td>
<td>0.012</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Lagrange multiplier</td>
<td>0.130</td>
</tr>
<tr>
<td>$\bar{\pi}$</td>
<td>Net income</td>
<td>0.641</td>
</tr>
</tbody>
</table>
B Solving for steady-state relations

B.1 Derivation of Equation (21)

Equation (16) is:

\[ \lambda_t \left[ \frac{1}{N_{t-1}} \left( \frac{K_{t-1}}{\kappa} - D_{t-1} \right) \right] = \beta \mathbb{E}_t \left\{ \lambda_{t+1} \left[ \frac{N_{t+1}}{N_t^2} \left( \frac{K_t}{\kappa} - D_t \right) + \psi \bar{d} \right] \right\}. \]

Its steady-state expression is as follows:

\[ \frac{1}{\bar{N}} \left( \frac{\bar{K}}{\kappa} - \bar{D} \right) = \beta \left[ \frac{1}{N} \left( \frac{\bar{K}}{\kappa} - \bar{D} \right) + \psi \bar{d} \right]. \] (B.1)

Equation (4) is:

\[ K_t = \kappa(D_t + P_t N_t). \]

It is expressed in the steady state as follows:

\[ \bar{K} = \kappa(\bar{D} + \bar{P} \bar{N}). \] (B.2)

Equations (B.2), and (B.1) imply the following relation:

\[ \bar{P} = \beta (\bar{P} + \psi \bar{d}). \] (B.3)

Now we can solve for \( \bar{P} \):

\[ \bar{P} = \frac{\beta \psi \bar{d}}{1 - \beta}. \] (B.4)

Thus, the steady-state dividend yield, \( \bar{r}^e \), is:

\[ \bar{r}^e = \frac{\bar{d}}{\bar{P}} = \frac{1 - \beta}{\psi \beta}. \]
B.2 Derivation of Equation (22)

Equation (B.2) implies that the steady-state value of equity, \(\bar{P} \bar{N}\), has the following form:

\[
\bar{P} \bar{N} = \frac{\bar{K}}{\kappa} - \bar{D}. \tag{B.5}
\]

Equation (B.5) implies that \(\bar{D} = \frac{\bar{K}}{\kappa} - \bar{P} \bar{N}\). If we combine it with Equation (B.4), we get:

\[
\bar{D} = \frac{\bar{K}}{\kappa} - \frac{\beta \psi \bar{d} \bar{N}}{(1 - \beta)}. \tag{B.6}
\]

Financial leverage in the steady state is defined as:

\[
\bar{L} = \frac{\bar{D}}{\bar{D} + \bar{P} \bar{N}}. \tag{B.7}
\]

Equations (B.5)-(B.7) imply the following relation:

\[
\bar{L} = 1 - \frac{\kappa \beta \psi \bar{d} \bar{N}}{\bar{K}(1 - \beta)}. \tag{B.8}
\]

B.3 Derivation of Equation (23)

To derive steady-state dividends, \(\bar{d}\), one needs to use Equations (3), (7), (8), and (B.6) as well as the definition of EBIT (Equation 6) in the steady state:

\[
\text{EBIT} = \bar{S} - \bar{C} - \bar{K} \delta. \tag{B.8}
\]

If \(\Phi_r = 0\), the equations imply the following relation:

\[
\bar{d} \bar{N} = \left[\text{EBIT} - \left(\frac{\bar{K}}{\kappa} - \frac{\beta \psi \bar{d} \bar{N}}{1 - \beta}\right)\right] r^*(1 - \tau). \tag{B.9}
\]
It leads to Equation (32):
\[ \bar{d} = \frac{\text{EBIT} - r^*\bar{K}}{\frac{N}{1-\tau} - \frac{\beta\psi N r^*}{1-\beta}}. \]

**B.4 Derivation of Equation (24)**

To derive steady-state capital stock, \( \bar{K} \), one needs to use Equations (14), (15), (17), and (B.8). If \( \Phi_r = 0 \), the equations imply the following relation:
\[ \frac{\alpha(1-\eta)}{K} \times \frac{\text{EBIT} + \bar{K} \delta}{\eta + \alpha(1-\eta)} - \delta - \frac{r^*}{\kappa} = 0. \]  
(B.10)

It leads to Equation (24):
\[ \bar{K} = \frac{\alpha(1-\eta)\text{EBIT}}{\frac{r^*}{\kappa}[\eta + \alpha(1-\eta)] + \delta\eta}. \]

**B.5 Derivation of Equation (25)**

To derive Equation (25), one needs to combine Equations (20)-(24) and (B.4):
\[
\text{WACC} = (1-\tau)r^* \left( 1 - \frac{\beta\kappa\psi\bar{N}}{K(1-\beta)} \times \frac{\text{EBIT} - r^*\bar{K}}{\frac{N}{1-\tau} - \frac{\beta\psi N r^*}{1-\beta} - \bar{d}} \right) + \frac{\beta\kappa\psi\bar{N}}{K(1-\beta)} \times \frac{1-\beta}{\beta\psi \rho^2} ,
\]
\[ = \frac{(1-\tau)\text{EBIT}}{\frac{K}{\kappa}}. \]

**B.6 Derivation of Equation (32)**

To derive steady-state dividend, \( \bar{d} \), one needs to use Equations (14), (15), (17), (B.6), and (B.8). If \( \Phi_r \neq 0 \), the equations imply the following relation:
\[ \frac{\alpha(1-\eta)}{K} \times \frac{\text{EBIT} + \bar{K} \delta}{\eta + \alpha(1-\eta)} - \delta - \frac{r^*(1 + \Phi_r)}{\kappa} + r^*\Phi_r\kappa \left[ \frac{\beta\psi\bar{N}}{(1-\beta)K} \right]^2 \bar{d}^2. \]  
(B.11)
It leads to Equation (32):

$$
\bar{d} = \frac{(1 - \beta)K}{\beta \psi N} \times \sqrt{\frac{\delta}{r^* \Phi r} \times \frac{\eta}{\eta + \alpha(1 - \eta)} + \frac{1 + \Phi r}{\Phi r \kappa^2} - \frac{\text{EBIT}}{r^* \Phi r} \times \frac{\alpha(1 - \eta)}{\eta + \alpha(1 - \eta)}}.
$$

**B.7 Capital stock, $\bar{K}$, in the steady state if there are bankruptcy costs**

By manipulating Equations (3), (7), (8), (32), (B.6), and (B.8), one derives that:

$$
\bar{K} = \frac{\Lambda_2 \Lambda_5 - 2\Lambda_3 \Lambda_4 \pm \sqrt{(2\Lambda_3 \Lambda_4 - \Lambda_5^2)^2 + 4(\Lambda_1 \Lambda_2^2 - \Lambda_3^2)\Lambda_4^2}}{2(\Lambda_1 \Lambda_2^2 - \Lambda_3^2)} \tag{B.12}
$$

where

$$
\Lambda_1 = \frac{\delta}{r^* \Phi r} \times \frac{\eta}{\eta + \alpha(1 - \eta)} + \frac{1 + \Phi r}{\Phi r \kappa^2}, \tag{B.13}
$$

$$
\Lambda_2 = \frac{1 - \beta}{(1 - \tau) \beta \psi} - r^*(1 + 2\Phi r), \tag{B.14}
$$

$$
\Lambda_3 = \frac{\delta \eta}{\eta + \alpha(1 - \eta)} + \frac{2r^*(1 + \Phi r)}{\kappa}, \tag{B.15}
$$

$$
\Lambda_4 = \frac{\text{EBIT}}{\eta + \alpha(1 - \eta)}, \tag{B.16}
$$

$$
\Lambda_5 = \frac{\text{EBIT}}{r^* \Phi r} \times \frac{\alpha(1 - \eta)}{\eta + \alpha(1 - \eta)}. \tag{B.17}
$$

The “$\pm$” sign in Equation (B.12) does not mean that there are two possible values for $\bar{K}$. One root is eliminated after manual verification which involves checking whether all the equations, defining the equilibrium, are satisfied.
References


